23.02.24

$$\frac{\dfrac{9(3x+8)^2}{2\sqrt{9x-4}} - 9\sqrt{9x-4}}{(3x+8)^4} = \frac{\dfrac{9(3x+8)^2 - 18(9x-4)}{2\sqrt{9x-4}}}{(3x+8)^4} = \frac{9(3x+8)^2 - 18(9x-4)}{2(3x+8)^4\sqrt{9x-4}}$$

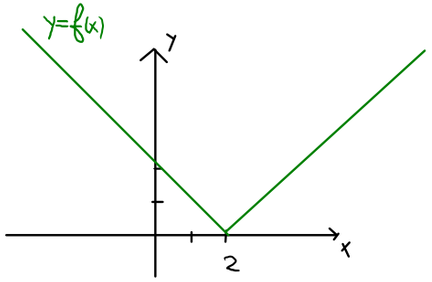
2.9.3 Les fonctions suivantes sont-elles dérivables en a ?

a) $f(x) = |x-2|$, $a = 2$ b) $f(x) = x\sqrt{|x|}$, $a = 0$

a) $ED(f) = \mathbb{R}$, $EC(f) = \mathbb{R}$

$f'(2)$ existe-t-il ?

$$f(x) = \begin{cases} -x+2 & \text{si } x < 2 \\ x-2 & \text{si } x \geq 2 \end{cases}$$



$$f'(x) = \begin{cases} -1 & \text{si } x < 2 \\ 1 & \text{si } x > 2 \end{cases}$$

$f'(2)$ n'existe pas !

b) $f(x) = x\sqrt{|x|}$, $a = 0$

$ED(f) = \mathbb{R}$, $EC(f) = \mathbb{R}$

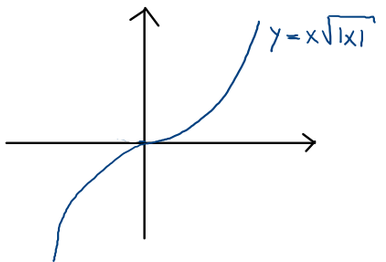
$$f(x) = \begin{cases} x\sqrt{-x} & \text{si } x < 0 \\ x\sqrt{x} & \text{si } x > 0 \end{cases} \qquad f'(x) = \begin{cases} \frac{3}{2}\sqrt{-x} & \text{si } x < 0 \\ \frac{3}{2}\sqrt{x} & \text{si } x \geq 0 \end{cases}$$

$x>0$: $\left(x\sqrt{x}\right)' = \left(x^1 \cdot x^{\frac{1}{2}}\right)' = \left(x^{\frac{3}{2}}\right)' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

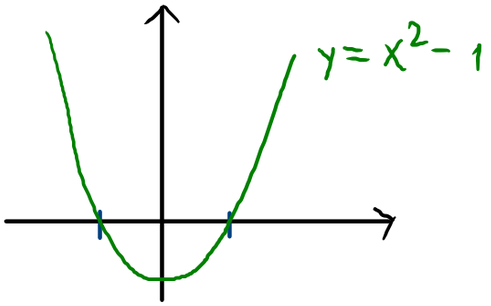
$\sqrt{x} + x\frac{1}{2\sqrt{x}} = \sqrt{x} + \frac{1}{2}\sqrt{x}$

$x<0$: $\left(x\sqrt{-x}\right)' = \sqrt{-x} + x\frac{1}{2\sqrt{-x}} \cdot (-1) = \sqrt{-x} + \underbrace{\frac{-x}{2\sqrt{-x}}}_{\frac{\sqrt{-x}}{2}} = \frac{3}{2}\sqrt{-x}$

f est dérivable partout et en particulier en 0



d) $f(x) = | x^2 - 1 | - 2 \ , \ a = -1$



$$f(x) = \begin{cases} x^2 - 3 & \text{si } x \leq -1 \\ -x^2 - 1 & \text{si } -1 < x < 1 \\ x^2 - 3 & \text{si } x \geq 1 \end{cases}$$

$$f'(x) = \begin{cases} 2x & \text{si } x < -1 \\ -2x & \text{si } -1 < x < 1 \\ 2x & \text{si } x > 1 \end{cases}$$

$f'(-1)$ n'existe pas.