28.09.23

4.2.1 Résoudre les équations ci-dessous :

e) $2^{-100x} = 0,5^{x-4}$

$$\boxed{\text{si } a>0,\ a^x = a^y \Leftrightarrow x = y}$$

$$2^{-100x} = \left(\frac{1}{2}\right)^{x-4}$$

$$2^{-100x} = \left(2^{-1}\right)^{x-4}$$

$$2^{-100x} = 2^{-x+4}$$

$$\Rightarrow \quad -100x = -x+4 \quad \Big|\ +x$$

$$-99x = 4 \quad \Big|\ \div(-99)$$

$$x = -\frac{4}{99}$$

$$S = \left\{\frac{-4}{99}\right\}$$

k) $3^{4x+2} - 36 \cdot 3^{2x+1} = -243$

$$3^{4x+2} - 36 \cdot 3^{2x+1} + 243 = 0$$

$$3^{2(2x+1)} - 36 \cdot 3^{2x+1} + 243 = 0$$

$$\left(3^{2x+1}\right)^2 - 36 \cdot 3^{2x+1} + 243 = 0$$

Posons $y = 3^{2x+1}$ (on fait un changement de variable)

$$y^2 - 36y + 243 = 0$$

$$(y - 27)(y - 9) = 0$$

Donc $\begin{cases} y = 27 & (1) \\ \text{ou} & \\ y = 9 & (2) \end{cases}$

(1)

$$3^{2x+1} = 27$$

$$3^{2x+1} = 3^3$$

$$\Rightarrow 2x+1 = 3$$

$$x = 1$$

(2)

$$3^{2x+1} = 9$$

$$3^{2x+1} = 3^2$$

$$\Rightarrow 2x+1 = 2$$

$$2x = 1$$

$$x = 1/2$$

$$S = \left\{ \frac{1}{2} ; 1 \right\}$$

l) $5 \cdot 5^{4x-7} - 120 \cdot 5^{2x-3} = 625$

$5^{4x-6} - 120 \cdot 5^{2x-3} - 625 = 0$

$(5^{2x-3})^2 - 120 \cdot 5^{2x-3} - 625 = 0$

$y = 5^{2x-3}$:

$y^2 - 120y - 625 = 0$

$(y-125)(y+5) = 0$

Donc $\begin{cases} y_1 = 125 \\ y_2 = -5 \end{cases}$

y_1 : $5^{2x-3} = 125$

$5^{2x-3} = 5^3$

$2x-3 = 3$

$2x = 6$

$x = 3$

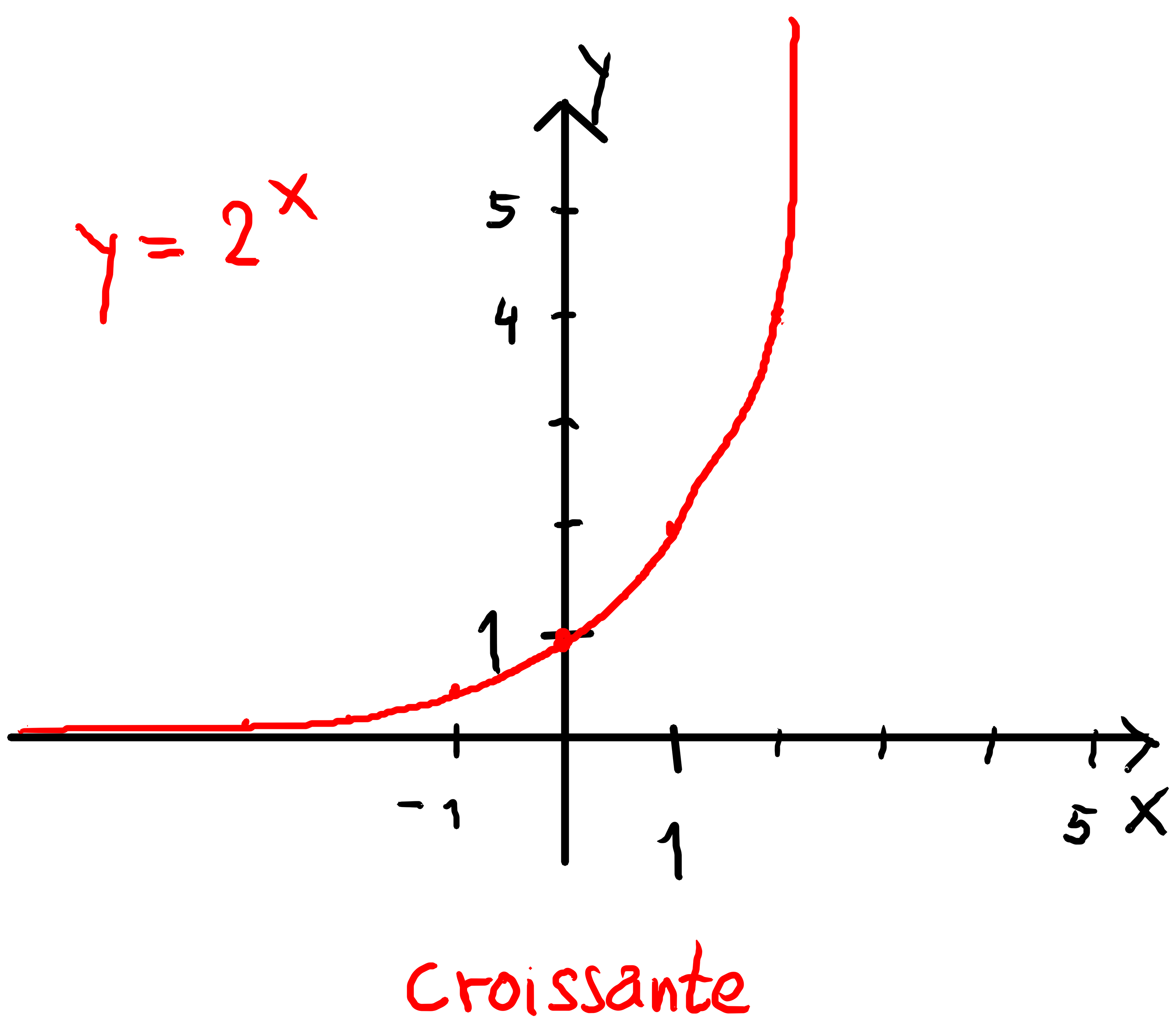
y_2 : $5^{2x-3} = -5$

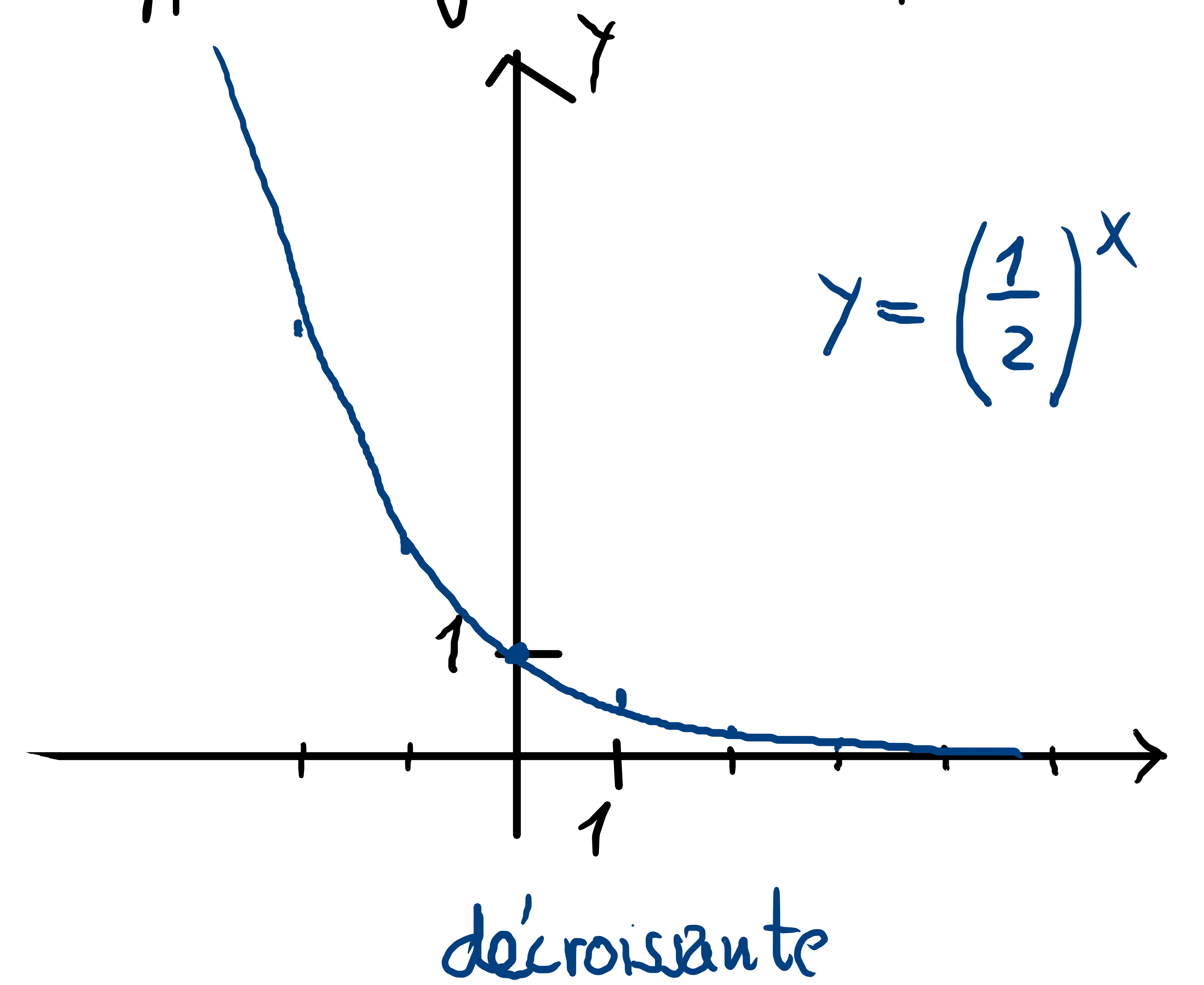
$\Rightarrow$ pas possible

$S = \{3\}$

Fonctions exponentielles

Soit $a > 0$, $a \neq 1$, $f(x) = a^x$ est appelée fonction exponentielle.





Comment résoudre $2^x = 10$?

- $2^x = 8 \Rightarrow x = 3$
- $2^x = 16 \Rightarrow x = 4$

On voit que $3 < x < 4$ dans $2^x = 10$.

Logarithmes

Soit $a > 0$, $a \neq 1$

$$\boxed{a^y = x \iff y = \log_a(x)}$$

Exemples

- $\log_2(2) = 1 \iff 2^1 = 2$
- $\log_2(8) = 3 \iff 2^3 = 8$
- $\log_2(1024) = 10 \iff 2^{10} = 1024$
- $\log_2(0{,}125) = -3 \iff 2^{-3} = 0{,}125$

$\log_{10}(1000) = 3 \iff 10^3 = 1000$

$\boxed{\text{TI30}}$ $\log_{10}(2023) \cong 3.305995882770805 \iff 10^{3.305995882770805} \cong 2023$

En fait $a^{\log_a(x)} = x$

$\log_a(a^y) = y$

4.2.2 Calculer à la main :

a) $\log_3(1)$ b) $\log_2(8)$ c) $\log_2(64)$ d) $\log_2(1'024)$

e) $\log_5(5)$ f) $\log_3(\sqrt{3})$ g) $\log_{243}(1/243)$ h) $\log_3(27)$

a) $\log_3(1) = 0 \iff 3^0 = 1$

c) $\log_2(64) = 6 \iff 2^6 = 64$

i) $\log(1'000) = \log_{10}(1000) = 3$

n) $\ln(e^2) = \log_e(e^2) = 2$ $e \cong 2.71$