31.08.23

1.2.2 Écrire les nombres complexes ci-dessous sous forme trigonométrique :

a) 1 Axe Re(z) $[1;0]$

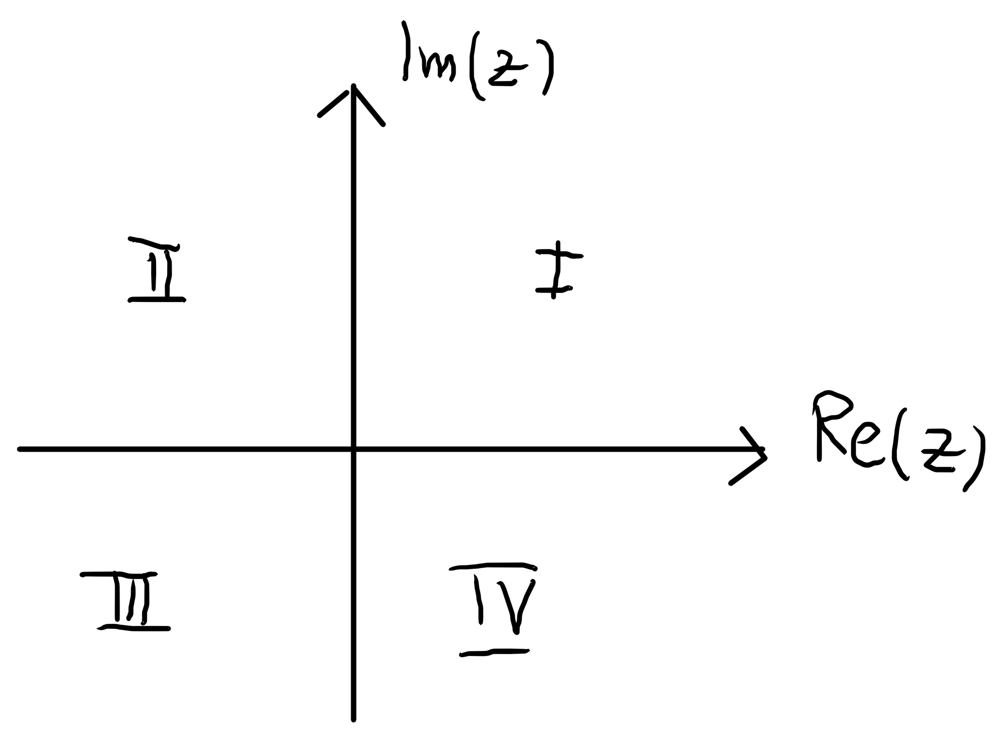
b) i Axe Im(z) $[1;\frac{\pi}{2}]$

c) -2 Axe Re(z) $[2;\pi]$

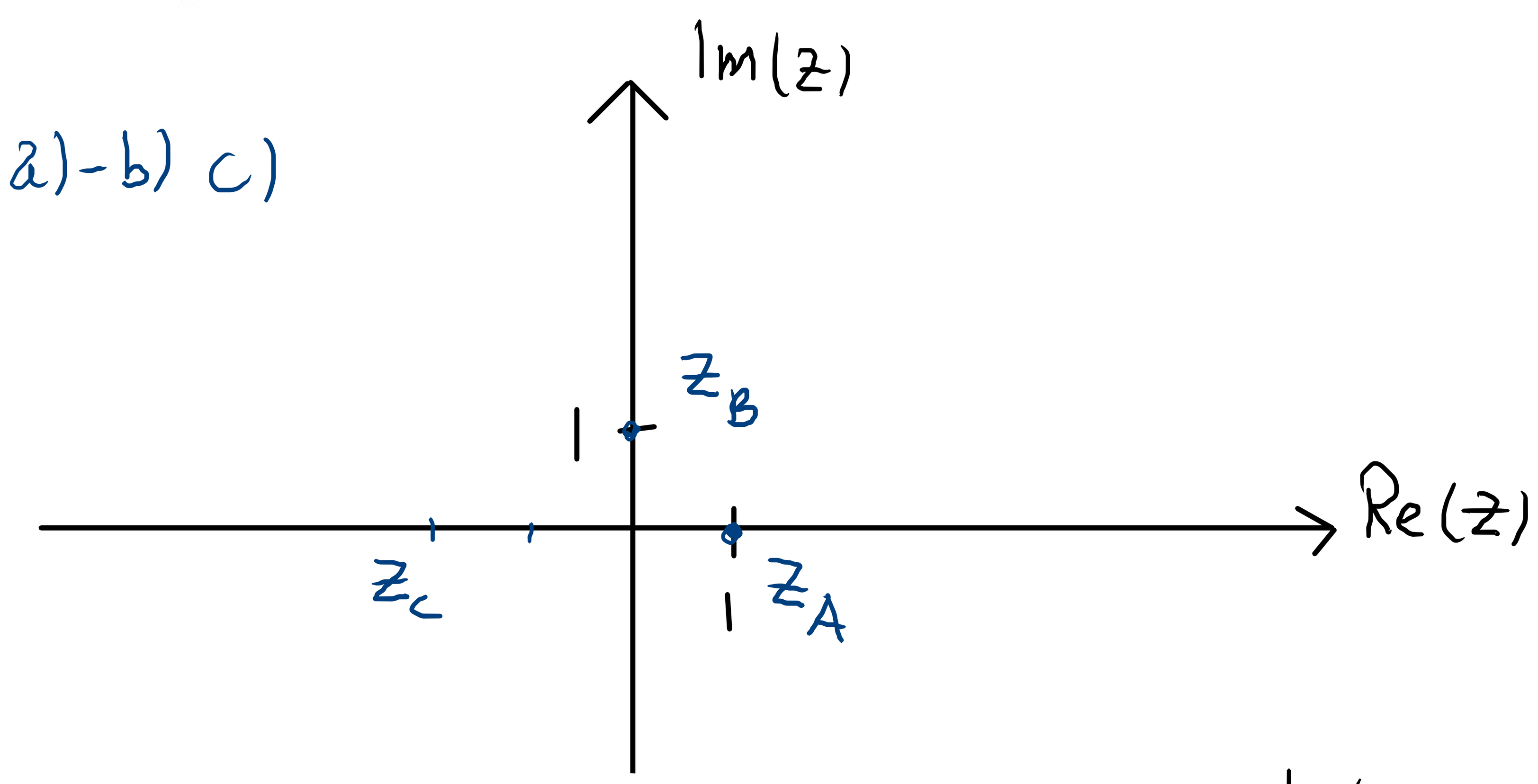
d) $-1-i$ III $(-1;-1)$

e) $-1-\sqrt{3}i$ III $(-1;-\sqrt{3})$

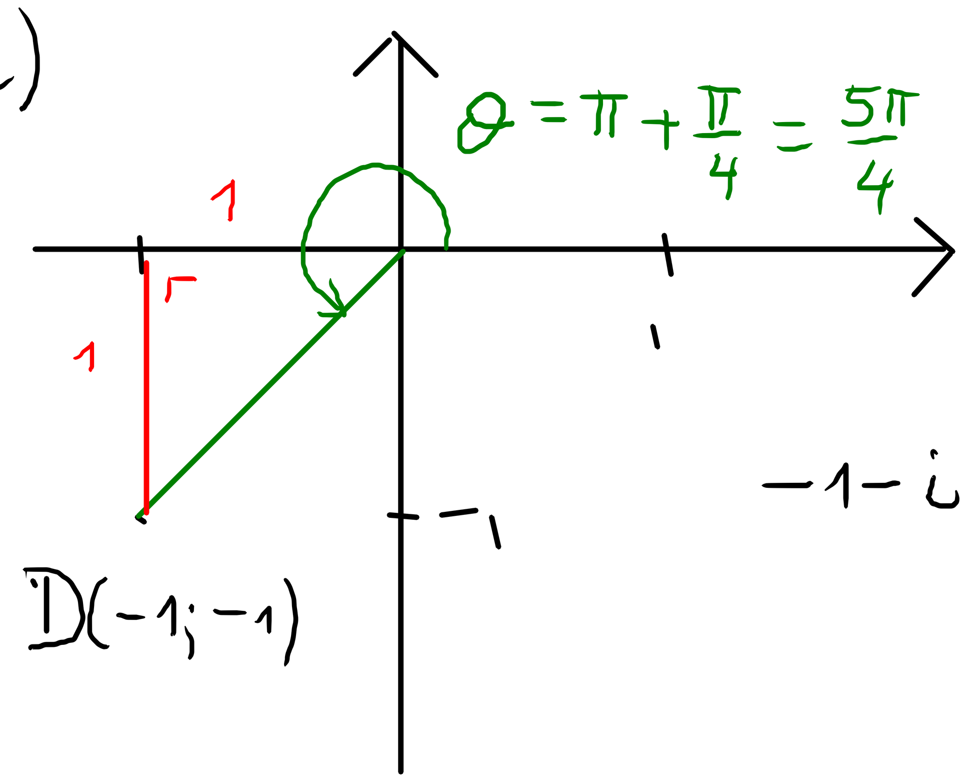
f) $3+4i$ I



a)-b) c)

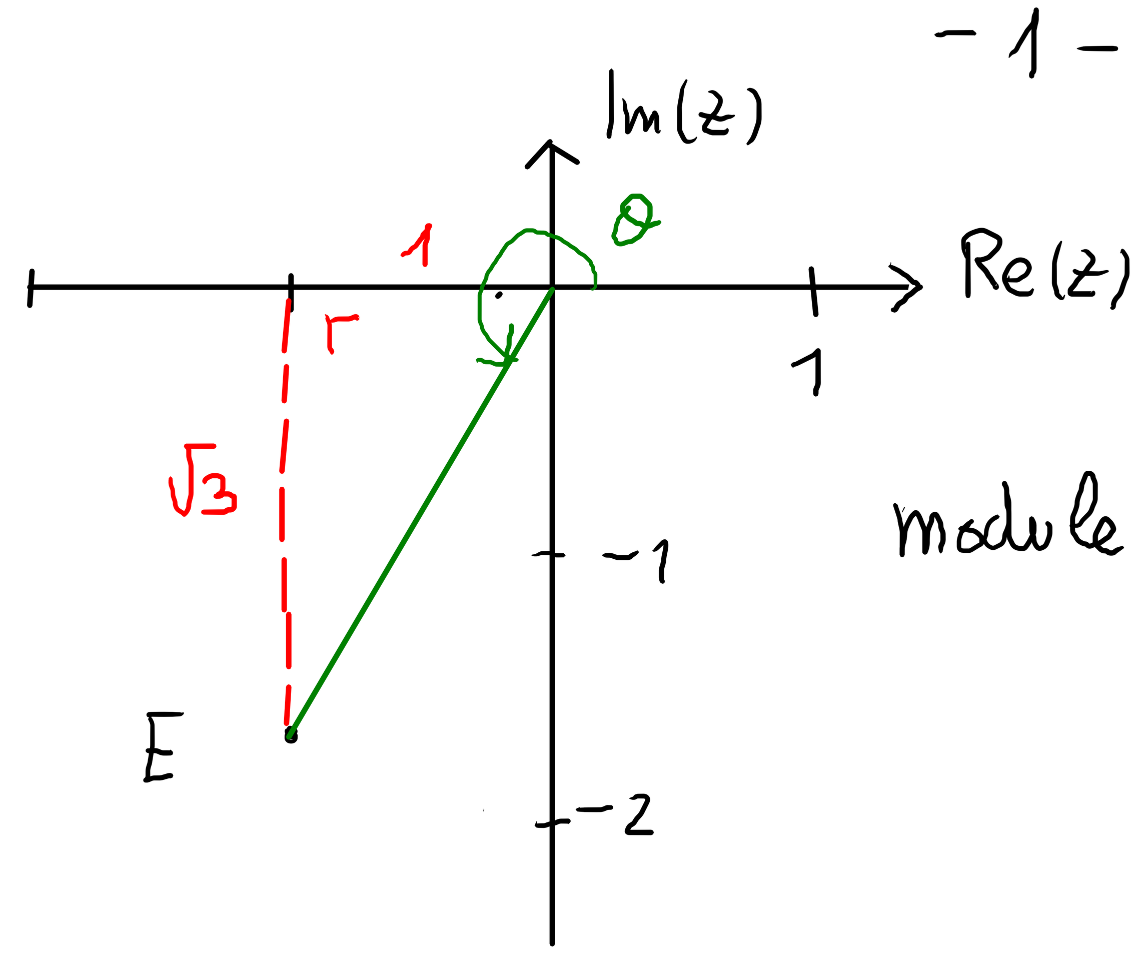


d)



$-1-i = [\sqrt{2} ; \frac{5\pi}{4}]$

e)

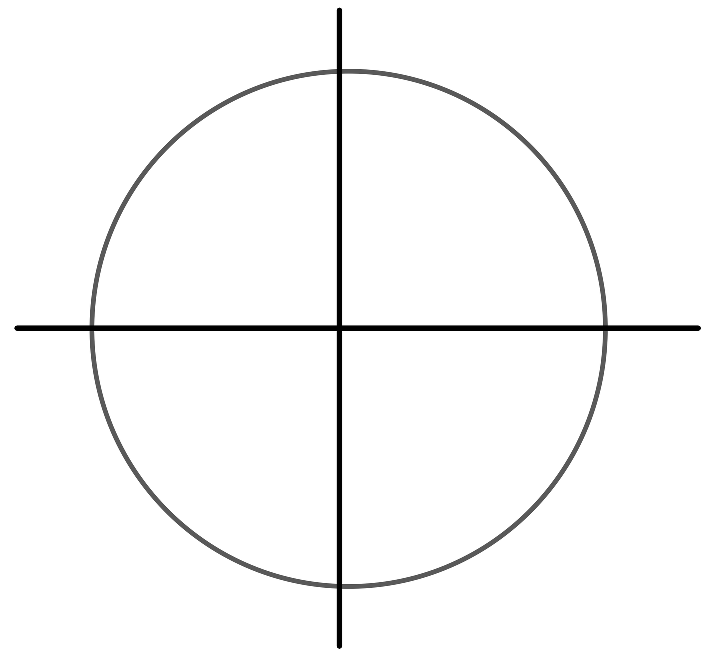


$-1-\sqrt{3}i = 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

module : $\sqrt{1^2+(\sqrt{3})^2} = 2$

$$\begin{cases} \cos\theta = -1/2 \\ \sin\theta = -\sqrt{3}/2 \end{cases}$$

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$



$$\begin{cases} \cos\theta = -1/2 \\ \sin\theta = -\sqrt{3}/2 \end{cases} \Rightarrow$$

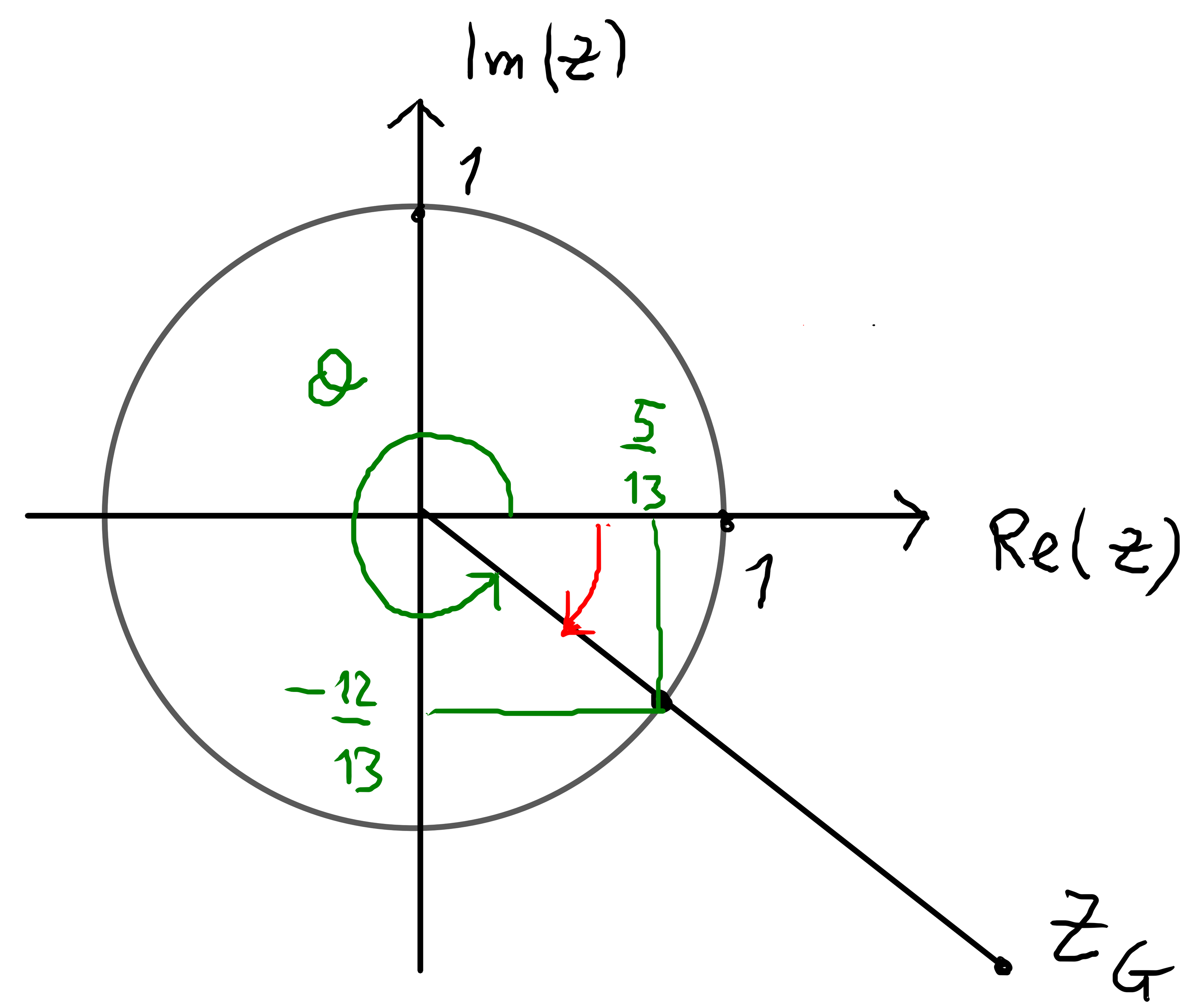
$\theta = 120°$ ou $-120°$

$-60° + \ldots = 180°$

$\theta = -60°$ ou $240° = 240 \cdot \frac{\pi}{180}$ $\frac{4\pi}{3}$

f) $3+4i = z_F$ $\qquad |z_F| = 5$

$$z_F = 5\left(\underset{\substack{\uparrow \\ \cos\theta}}{\frac{3}{5}} + \underset{\substack{\uparrow \\ \sin\theta}}{\frac{4}{5}} i\right) = [5 ; 0.927295218001612]$$



g) $5 - 12i = z_G$

$|z_G| = 13$

$$z_G = 13\left(\frac{5}{13} - \frac{12}{13} i\right)$$

$$\begin{cases} \cos\theta = \dfrac{5}{13} \Rightarrow \theta = 1.176005207095135 \text{ ou } \theta = -1.176005207095135 \\ \sin\theta = -\dfrac{12}{13} \Rightarrow \theta = -1.176005207095135 \text{ ou } \theta = 4.317597860684928 \end{cases}$$

$$z_G = [13 ; 5.107180307179586]$$

1.2.3 Écrire les nombres complexes ci-dessous sous forme algébrique :

a) $\left[4; -\frac{\pi}{3}\right]$

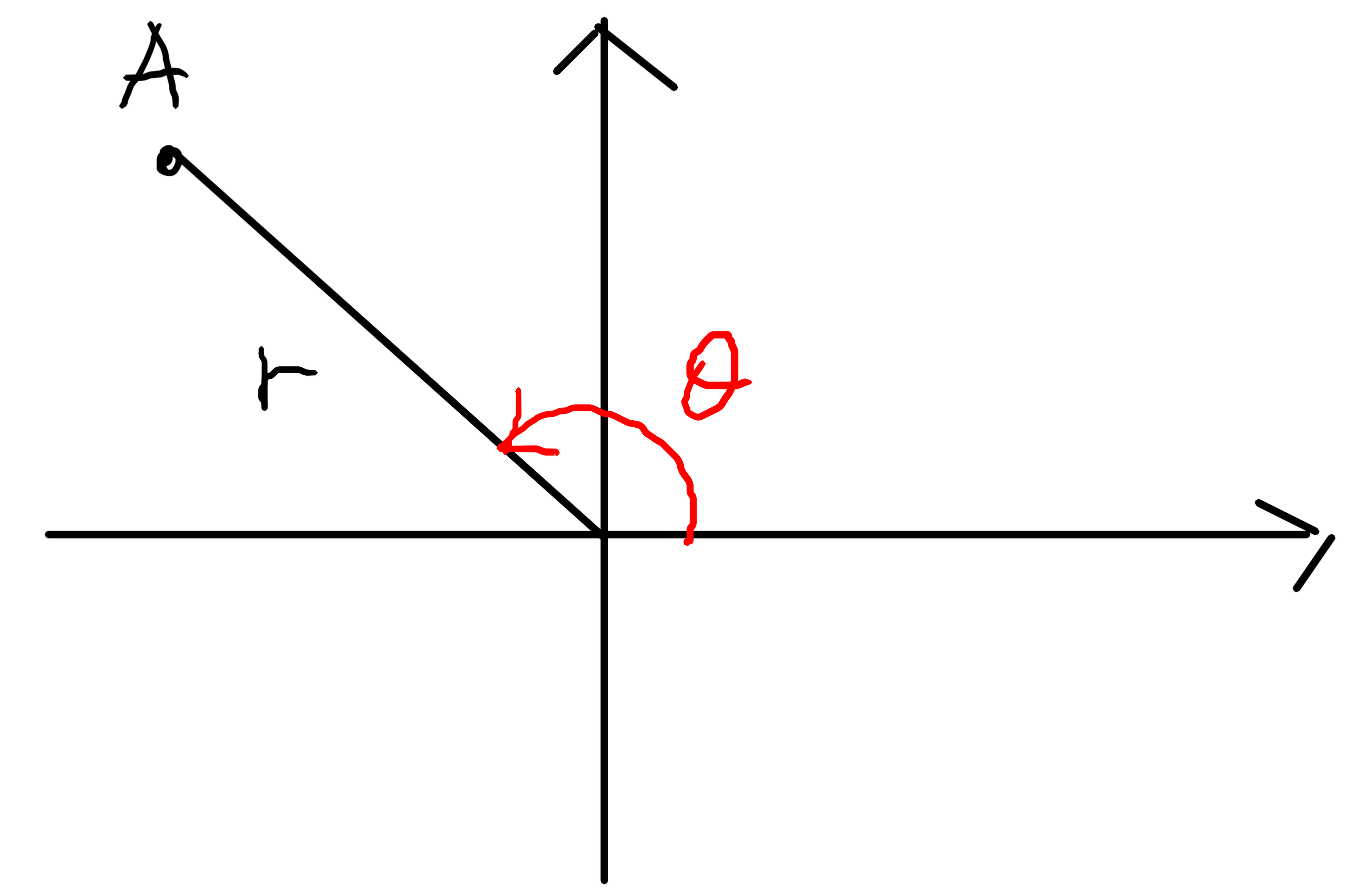
b) $\left[\frac{3}{4}; \frac{3\pi}{4}\right]$

c) $[\pi; -\pi]$

d) $\left[4; \frac{\pi}{3}\right]$

e) $\left[1; -\frac{\pi}{2}\right]$

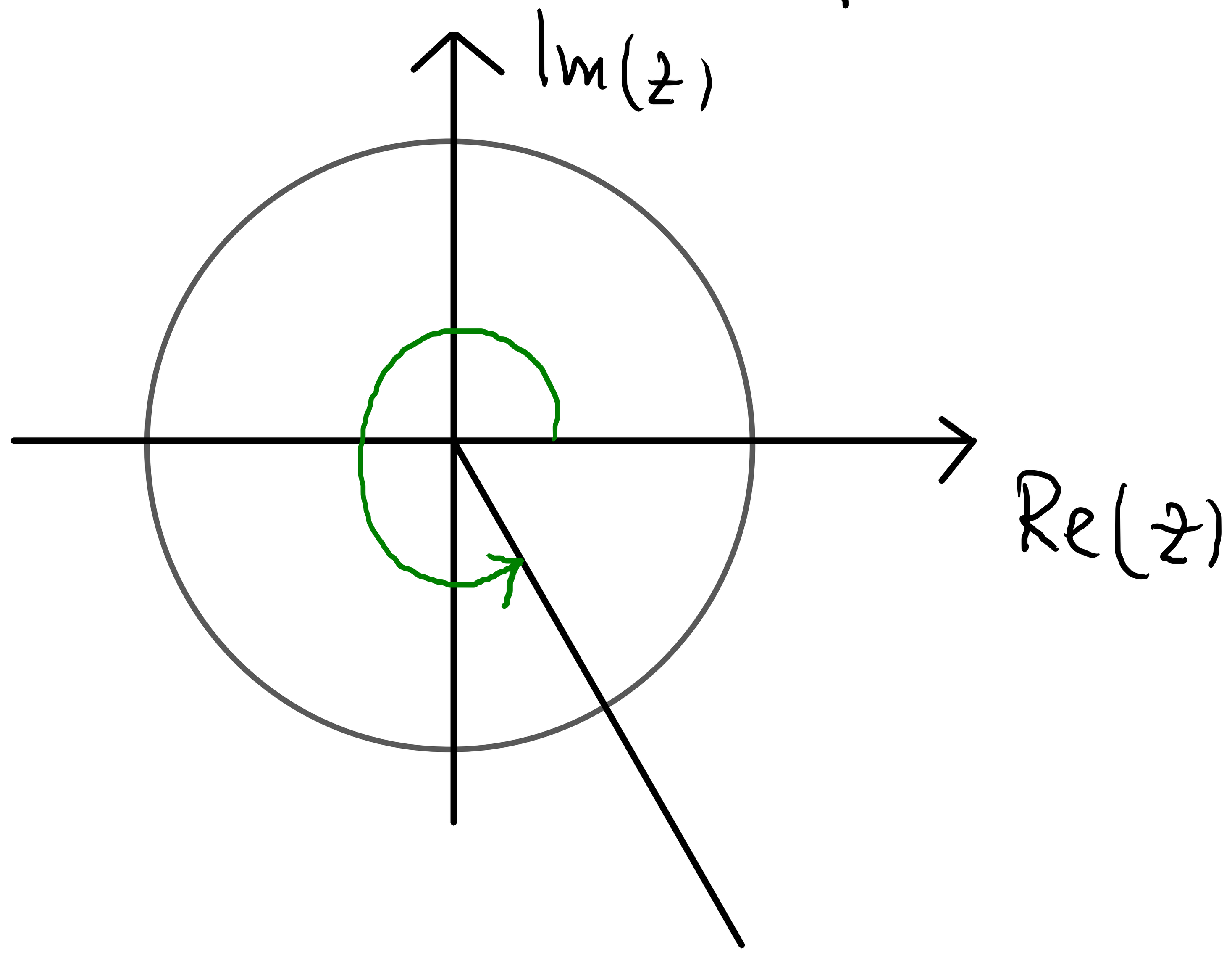
f) $e^{i\pi}$ plus tard



$z_A = a + bi$, $|z_A| = r$ argument θ

$z_A = [r, \theta] = r(\cos\theta + \sin\theta\, i)$

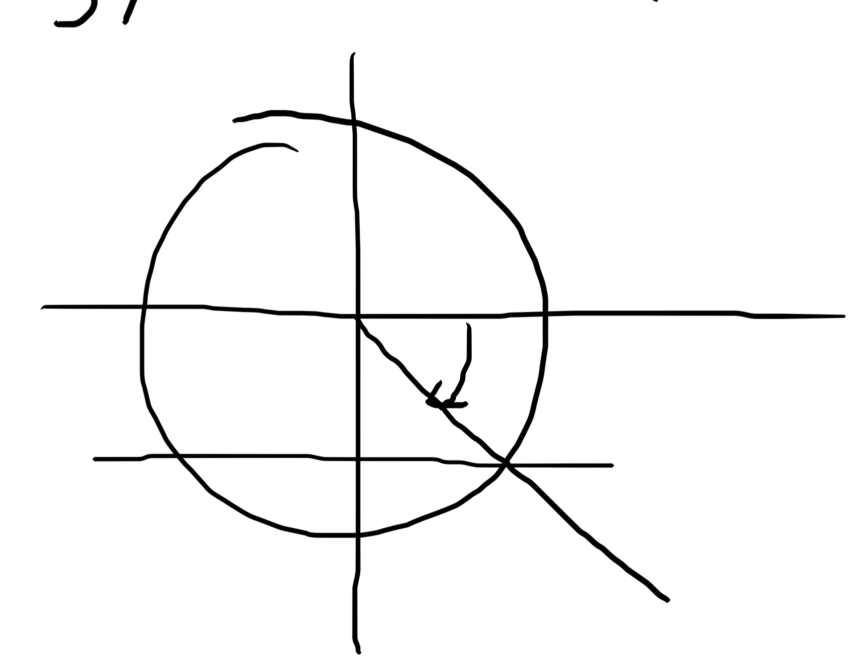
a) $z_A = 4\left(\cos\left(-\frac{\pi}{3}\right) + \sin\left(-\frac{\pi}{3}\right) i\right) = 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = 2 - 2\sqrt{3}\, i$



$\theta = 300°$

$\cos\left(-\frac{\pi}{3}\right) = \cos(-60°) = \cos(60°) = \frac{1}{2}$

$\sin\left(-\frac{\pi}{3}\right) = \sin(-60°) = -\sin(60°) = -\frac{\sqrt{3}}{2}$

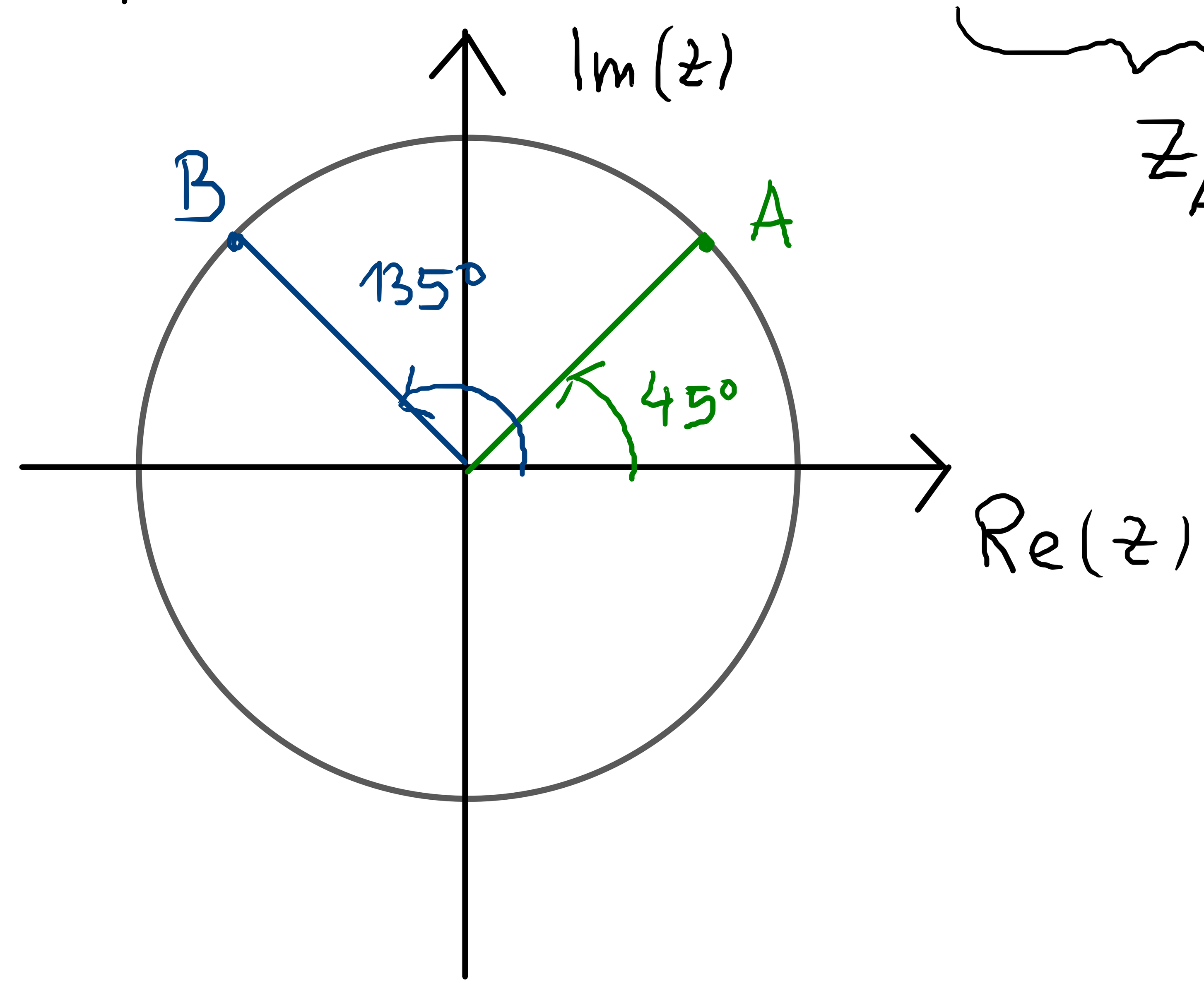


- $\cos(-\alpha) = \cos(\alpha)$
- $\sin(-\alpha) = -\sin(\alpha)$

1.2.4 Calculer :

a) $\left[2;\frac{\pi}{4}\right]\cdot\left[3;\frac{\pi}{6}\right]$ b) $\left[6;\frac{2\pi}{3}\right]:\left[3;-\frac{\pi}{3}\right]$ c) $\left[2;\frac{\pi}{3}\right]^3$

d) plus simple $\underbrace{\left[1;\frac{\pi}{4}\right]}_{z_A}\cdot\underbrace{\left[1;\frac{3\pi}{4}\right]}_{z_B} = [1; 45°]\cdot[1; 135°]$



$$= \left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$$

$$= \frac{\sqrt{2}}{2}(1+i)\cdot\frac{\sqrt{2}}{2}(-1+i)$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}(1+i)(-1+i)$$

$$= \frac{1}{2}\left(-1+i^2\right) = \frac{1}{2}\cdot(-2) = -1$$

Donc $\left[1;\frac{\pi}{4}\right]\cdot\left[1;\frac{3\pi}{4}\right] = [1;\pi]$

$+$ $\times$

En fait $\boxed{[r_1,\theta_1]\cdot[r_2,\theta_2] = [r_1\cdot r_2\,,\ \theta_1+\theta_2]}$