

e) $n=1$

$$\sum_{k=1}^1 \frac{1}{(2k-1)(2k+1)} = \frac{1}{(2-1)(2+1)} = \frac{1}{3}$$

$$= \frac{1}{2+1} \quad \checkmark$$

$n \checkmark \Rightarrow n+1 \checkmark$

$$\sum_{k=1}^{n+1} \frac{1}{(2k-1)(2k+1)}$$

$$= \sum_{k=1}^n \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2(n+1)-1)(2(n+1)+1)}$$

hyp. de réc.

$$= \frac{n}{2n+1} + \frac{1}{(2n+1)(2n+3)}$$

$$= \frac{n \cdot (2n+3) + 1}{(2n+1)(2n+3)} = \frac{2n^2 + 3n + 1}{(2n+1)(2n+3)}$$

$$= \frac{\cancel{(2n+1)}(n+1)}{\cancel{(2n+1)}(2n+3)} = \frac{(n+1)}{(2 \cdot (n+1) + 1)}$$

CQFD