Exercice 1

Calculer l'équation de la tangente au graphe $y = f(x)$ au point d'abscisse $x = a$.

a) $f(x) = 1 - x\,e^x$, $a = 0$.

b) $f(x) = x^5 - 2\,x + 1$, $a = -1$.

Burier

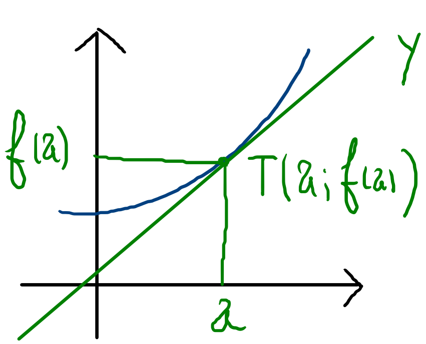
Équation de la tangente t à la courbe $y = f(x)$ au point

$T(a\,;f(a))$: $\boxed{y - f(a) = f'(a) \cdot (x - a)}$

$$(e^x)' = e^x \quad ; \quad (e^u)' = u' \cdot e^u$$

$$(\ln(x))' = \frac{1}{x} \quad ; \quad (\ln(u))' = \frac{u'}{u}$$

a)



$$y = \underbrace{f'(a)}_{\text{pente}}\,x + h \quad ; \quad f(a) = f'(a)\,a + h$$

$$h = f(a) - f'(a)\,a$$

$$y = f'(a)\,x + f(a) - f'(a)\,a$$

$$\boxed{y = (x-a)\,f'(a) + f(a)}$$

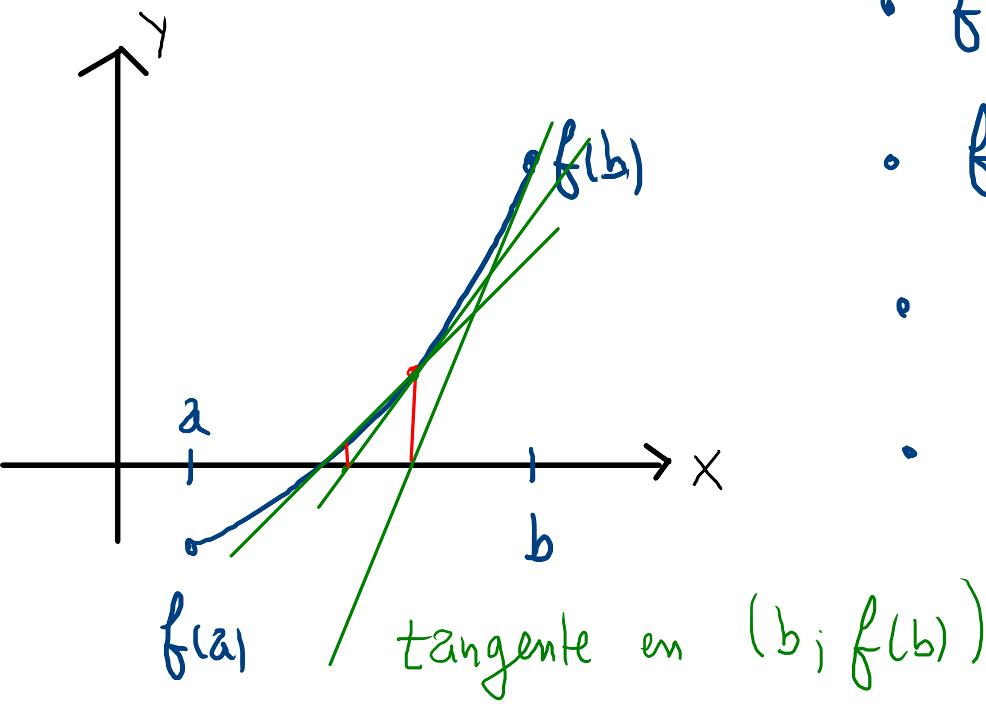
a) $f(x) = 1 - \underbrace{x}_{u}\,\underbrace{e^x}_{v}, \; a = 0.$

$$(uv)' = u'v + uv'$$

$$f'(x) = 0 - \left(1 \cdot e^x + x\,e^x\right)$$

$$= -e^x\,(1 + x)$$

La méthode de Newton



- f est différentiable
- $f(a) \cdot f(b) < 0$
- f strictement croissante
- $f''(x) > 0$

Exercice 2

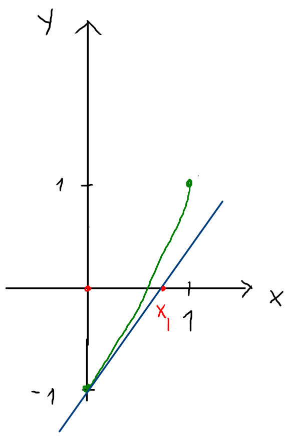
10^{-3}

Trouver ~~aux dix~~ millième près la racine de chacune des équations suivantes avec la méthode de Newton en partant du point x_0 donné.

a) $f(x) = x(1+e^x) - e^x$, $x_0 = 0$.

CRM

$$f'(x) = 1\cdot(1+e^x) + x\cdot e^x - e^x$$
$$= 1+e^x + xe^x - e^x = 1 + xe^x$$



$f(1) = 1(1+e) - e = 1$

0) $x_0 = 0$

tangente : $y = x - 1$; $\begin{cases} y = 0 \\ y = x-1 \end{cases} \Rightarrow x_1 = 1$

1) $x_1 = 1$

tangente : $y = 3{,}7182\,(x-1) + 1$

$y = 3{,}7182\,x - 2{,}7182 \Rightarrow x_2 = \dfrac{2{,}7182}{3{,}7182} \cong 0{,}7311$

2) $x_2 = 0{,}7311$

tangente : $y = f'(0{,}7311)(x - 0{,}7311) + f(0{,}7311)$

Tangente t en x_0 $\quad y = f'(x_0)(x - x_0) + f(x_0)$

$\Rightarrow x_3 \cong 0{,}6626$

3) $x_3 = 0{,}6626$

tangente : $y = f'(0{,}6626)(x - 0{,}6626) + f(0{,}6626)$

$x_4 \cong 0{,}6591$

4) $x_4 = 0{,}6591$

tangente $y = f'(0{,}6591)(x - 0{,}6591) + f(0{,}6591)$

$x_5 = 0{,}6590$

D'où la solution $r = 0{,}659$

Equation de la tangente en $(x_n, f(x_n))$

$$y = f'(x_n)(x - x_n) + f(x_n)$$

$$y = 0 \Rightarrow \quad 0 = f'(x_n) \cdot x - f'(x_n) \cdot x_n + f(x_n)$$

$$f'(x_n)\, x = f'(x_n) \cdot x_n - f(x_n) \qquad \text{si } f'(x_n) \neq 0$$

$$x = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$