Sommes

$a = 3$, $b = 5$, $c = -2$, $d = 8$

Python : `l = [3, 5, -2, 8]`

`l[0] = 3` ; `l[2] = -2` ; `len(l) = 4`

somme de l :

```
l = [3, 5, -2, 8]
s = 0
for k in range(3):
        s = s + l[k]
print("somme de l", s)
```

$l = [3, 5, -2, 8]$

$l[0] \quad l[1] \quad l[2] \quad l[3]$

$l_0 = 3, \quad l_1 = 5, \quad l_2 = -2, \quad l_3 = 8$

$$\boxed{\sum_{K=0}^{3} l_K = l_0 + l_1 + l_2 + l_3}$$

1.1.1 On donne

$$x_1 = 3, \quad x_2 = 5, \quad x_3 = 6, \quad x_4 = 2 \quad \text{et} \quad x_5 = 7.$$

Calculer :

a) $\sum_{i=1}^{5} x_i$

b) $\sum_{i=2}^{4} x_i$

c) $\sum_{k=1}^{5} x_k$

d) $\sum_{j=1}^{5} {x_j}^3$

e) $\sum_{i=1}^{5} (x_i + 8) = \sum_{i=1}^{5} x_i + \sum_{i=1}^{5} 8$

f) $\sum_{k=1}^{5} (8 \cdot x_k)$

Propriétés

Soit deux suites de nombres : $(a_k)_{k=0}^{n}$ et $(b_j)_{j=0}^{n}$

① $$\sum_{i=0}^{n} (a_i + b_i) = (a_0 + b_0) + (a_1 + b_1) + \cdots + (a_n + b_n)$$
$$= \underbrace{a_0 + a_1 + \cdots + a_n}\, + b_0 + b_1 + \cdots + b_n$$
$$= \sum_{i=0}^{n} a_i + \sum_{i=0}^{n} b_i$$

② $$\sum_{k=0}^{n} m\, a_k = m a_0 + m a_1 + \cdots + m a_n = m(a_0 + \cdots + a_n)$$
$$= m \sum_{k=0}^{n} a_k \quad \text{où } m \in \mathbb{R}$$

③ $$\sum_{k=0}^{n} m = (n+1) \cdot m$$

$$\sum_{i=0}^{5} 8 = 8 + 8 + 8 + 8 + 8 + 8 = 6 \cdot 8 = 48$$

④ $$\sum_{i=1}^{3} (a_i + b_i)^2 \neq \sum_{i=1}^{3} a_i^2 + \sum_{i=1}^{3} b_i^2 = \sum_{i=1}^{3} (a_i^2 + b_i^2)$$
$$\parallel$$
$$\sum_{i=1}^{3} (a_i^2 + 2 a_i b_i + b_i^2)$$

⑤ $$\sum_{i=0}^{n} a_i b_i \neq \left(\sum_{i=0}^{n} a_i\right) \cdot \left(\sum_{i=0}^{n} b_i\right)$$

1.1.3 Développer les sommes suivantes

a) $\sum_{i=0}^{n}(-1)^i$ b) $\sum_{j=0}^{n} 3j$ c) $\sum_{k=1}^{n} n$ d) $\sum_{i=1}^{n} \frac{n}{i}$

a) • $n=0$ $\sum_{i=0}^{0} (-1)^i = (-1)^0 = 1$

• $n=1$ $\sum_{i=0}^{1} (-1)^i = (-1)^0 + (-1)^1 = -1+1 = 0$

• $n=2$ $\sum_{i=0}^{2} (-1)^i = (-1)^2 = 1$

Si n est pair : $\sum_{i=0}^{n} (-1)^i = 1$

Si n est impair : $\sum_{i=0}^{n} (-1)^i = 0$

c) $\sum_{j=1}^{n} n = n+n+\dots+n = n \cdot n = n^2$

d) $\sum_{i=1}^{n} \frac{n}{i} = n + \frac{n}{2} + \dots + \frac{n}{n} = n\left(1+\frac{1}{2}+\dots+\frac{1}{n}\right)$

b) $\sum_{j=0}^{n} 3j = 3 \cdot \sum_{j=0}^{n} j = 3 (\underbrace{0 + 1 + 2 + \dots + n}_{S_1})$

$$S_1 = 1 + 2 + 3 + \dots + n$$

$$S_1 = n + n-1 + n-2 + \dots + 1$$

$$2S_1 = (n+1) + (n+1) + \dots + (n+1) = n(n+1)$$

$$\boxed{S_1 = \frac{n(n+1)}{2}}$$

Question : $S_2 = \sum_{j=1}^{n} j^2 = 1^2 + 2^2 + \dots + n^2$