29.08.23

Nous avons vu que $S_1 = \sum_{K=1}^{n} K = \frac{n(n+1)}{2}$

Déterminons $S_2 = \sum_{K=1}^{n} K^2 = 1^2 + 2^2 + 3^2 + \dots + n^2$

Pour cela calculons

$$
\begin{aligned}
(n+1)^3 &= n^3 + 3n^2 + 3n + 1 \\
n^3 &= (n-1)^3 + 3(n-1)^2 + 3(n-1) + 1 \\
(n-1)^3 &= (n-2)^3 + 3(n-2)^2 + 3(n-2) + 1 \\
&\vdots \\
2^3 &= 1^3 + 3\cdot 1 + 3\cdot 1 + 1 \\
1^3 &= 1 \\
\hline
(n+1)^3 + \cancel{S_3} &= \cancel{S_3} + 3S_2 + 3S_1 + (n+1)
\end{aligned}
$$

$$3S_2 = (n+1)^3 - 3S_1 - (n+1)$$

$$3S_2 = (n+1)^3 - 3\frac{n(n+1)}{2} - (n+1)$$

$$3S_2 = (n+1)\left[(n+1)^2 - \frac{3}{2}n - 1\right]$$

$$3S_2 = (n+1)\left[n^2 + 2n \cancel{+1} - \frac{3}{2}n\right]$$

$$3S_2 = (n+1)\left[\frac{2n^2 + 4n - 3n}{2}\right]$$

$$3S_2 = (n+1)\frac{2n^2 + n}{2}$$

$$3S_2 = (n+1)\frac{n(2n+1)}{2}$$

$$\boxed{S_2 = \frac{n(n+1)(2n+1)}{6}}$$

Ex : calculer S_3

Triangle de Pascal

$(a+b)^0 = 1$

$(a+b)^1 \qquad \cancel{1} \qquad \cancel{1}$

$(a+b)^2 \qquad 1\,a^2 \qquad \cancel{2}\,ab \qquad 1\,b^2$

$(a+b)^3 \qquad 1\,a^3 \qquad 3\,a^2b \qquad 3\,ab^2 \qquad 1\,b^3$

$(a+b)^4 \qquad 1\,a^4 \qquad 4\,a^3b \qquad 6\,a^2b^2 \qquad 4\,ab \qquad 1\,b^4$

1.1.5

c) $$\sum_{k=-n}^{n}(k+1) = (-n+1) + (-n+2) + (-n+3) + \dots + (n-2) + (n-1) + n + (n+1)$$

$$= n + n + 1 = 2n + 1$$

$-n, -n+1, -n+2, \dots, 0, \dots, n, n+1$

$$\sum_{K=-3}^{3}(K+1) = \cancel{-2} + \cancel{(-1)} + 0 + \cancel{1} + \cancel{2} + 3 + 4$$

K: -3 -2 -1 0 1 2 3

$$\sum_{K=-n}^{n}(K+1) = 2n+1$$

1.1.8 Ecrire les sommes suivantes à l'aide du symbole Σ.

a) $2+4+6+\cdots 248=$

b) $1000+1010+1020+\cdots+1540=$

c) $1^2+2^2+3^2+\cdots 15^2=$

d) $1+2+4+8+16+\cdots+1024=$

e) $2+3+5+9+17+\cdots+1025=$

f) $4+12+36+108+324=$

g) $9-12+15-18+\cdots+303=$

h) $45-40+35-30+25-20+15=$

g) $9-12+15-18+\cdots+303 = \sum_{K=3}^{101} 3\cdot K \cdot (-1)^{K+1} = \sum_{K=3}^{101} (-1)^{K+1} 3K$

$3\cdot\cancel{3} - 3\cdot 4 + 3\cdot\cancel{5} + \cdots + 3\cdot(\cancel{101})$

$\rightarrow$ 1.1.8

$\boxed{\begin{matrix}1.1.13\\1.1.14\end{matrix}}$