1.1.13

c) $f(x) = (x^2 - 4x + 4)e^x = (x-2)^2 \cdot e^x$

① $ED(f) = \mathbb{R}$

② Parité : $f(-x) = \left((-x)^2 - 4(-x) + 4\right) e^{-x}$

$= (x^2 + 4x + 4) e^{-x}$

$= (x+2) e^{-x}$

Ni paire, ni impaire

③ Signe de $f(x)$ :

| $x$ | | 2 | |
|---|---|---|---|
| $(x-2)^2$ | $+$ | 0 | $+$ |
| $e^x$ | $+$ | | $+$ |
| $f(x)$ | $+$ | 0 | $+$ |

④ AV : aucune

AHD : $\lim\limits_{x \to +\infty} (x-2)^2 e^x = +\infty \cdot +\infty = +\infty$

aucune AH à droite

AHG : $\lim\limits_{x \to -\infty} (x-2)^2 e^x \overset{\text{IND}}{=} \text{“} +\infty \cdot 0 \text{”}$

$= \lim\limits_{x \to -\infty} \frac{(x-2)^2}{e^{-x}} \underset{\text{“}\frac{+\infty}{+\infty}\text{”}}{\overset{\text{BH}}{=}} \lim\limits_{x \to -\infty} \frac{2(x-2)}{-e^{-x}} \underset{\text{“}\frac{-\infty}{-\infty}\text{”}}{\overset{\text{BH}}{=}} \lim\limits_{x \to -\infty} \frac{2}{e^{-x}} \underset{\frac{2}{+\infty}}{=} 0$

Il y a une AHG $y = 0$.

⑤ <u>Croissance :</u>

$$f(x) = \underbrace{(x-2)^2}_{u}\,\underbrace{e^x}_{v}$$

$u = (x-2)^2 \quad ; \quad u' = 2(x-2)$

$v = e^x \quad ; \quad v' = e^x$

$$f'(x) = 2(x-2)e^x + (x-2)^2 e^x = (x-2)e^x\left(2 + x - 2\right)$$

$$= x(x-2)e^x$$

zéros de $f'(x)$ : $0$ et $2$

Signe de $f'(x)$ :

| $x$ | | $0$ | | $2$ | |
|---|---|---|---|---|---|
| $x$ | $-$ | $0$ | $+$ | | $+$ |
| $x-2$ | $-$ | | $-$ | $0$ | $+$ |
| $e^x$ | $+$ | | $+$ | | $+$ |
| $f'(x)$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $f(x)$ | ↗ | $\boxed{\text{max}}$ | ↘ | $\boxed{\text{min}}$ | ↗ |

max $(0 \,;\, 4)$

min $(2 \,;\, 0)$

$f(0) = 4 \cdot e^0 = 4$

$f(2) = 0$

⑥ <u>Courbure :</u>

$(x-2)^2 e^x \,,\; x = 2$

$$f'(x) = \underbrace{x(x-2)}_{u}\,\underbrace{e^x}_{v}$$

$u = x^2 - 2x \quad ; \quad u' = 2x - 2$

$v = e^x \quad ; \quad v' = e^x$

$$f''(x) = (2x-2)e^x + (x^2-2x)e^x$$
$$= (2x - 2 + x^2 - 2x)e^x = (x^2-2)e^x$$

zéros de $f''(x)$ : $x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

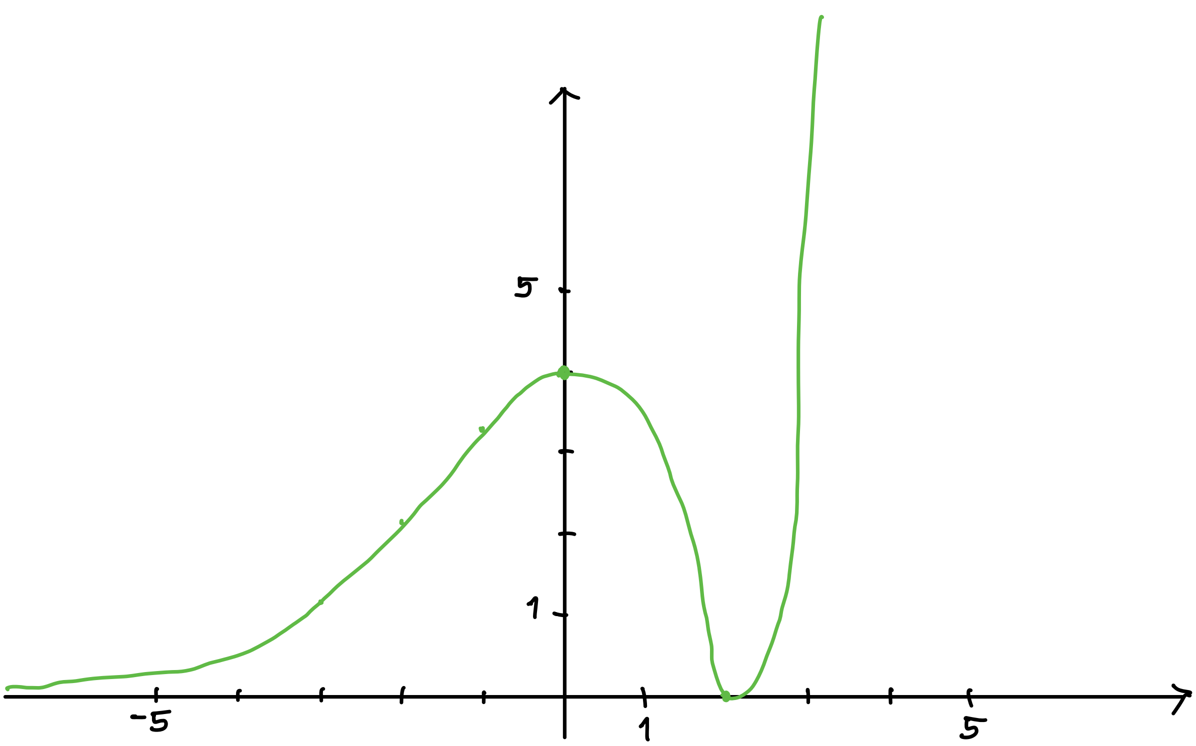
| $x$ | | $-\sqrt{2}$ | | $\sqrt{2}$ | |
|---|---|---|---|---|---|
| $(x^2-2)$ | + | 0 | − | 0 | + |
| $e^x$ | + | | + | | + |
| $f''(x)$ | + | 0 | − | 0 | + |
| $f(x)$ | ∪ | pi | ∩ | pi | ∪ |

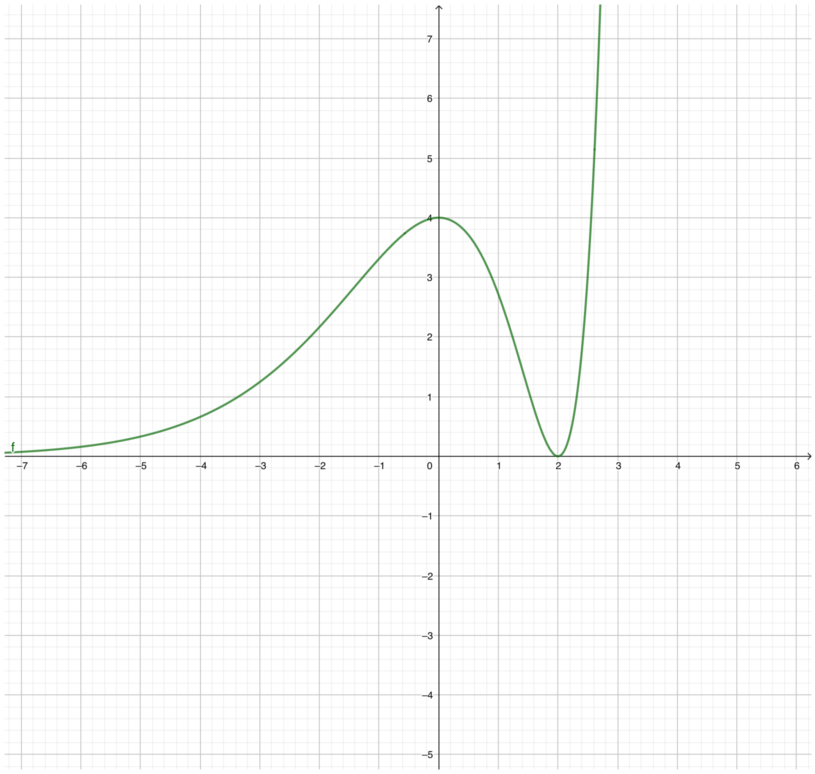
pi : $(-\sqrt{2} \,;\, 2{,}8)$

pi : $(\sqrt{2} \,;\, 1{,}4)$

$f(-\sqrt{2}) \cong 2{,}8$

$f(\sqrt{2}) = 1{,}4$





f
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
6
5
4
3
2
1
-1
-2
-3
-4
-5