Ex 1.1.14 d

$$f(x) = \ln\left(\frac{2x}{x+1}\right)$$

① **ED(f)** :

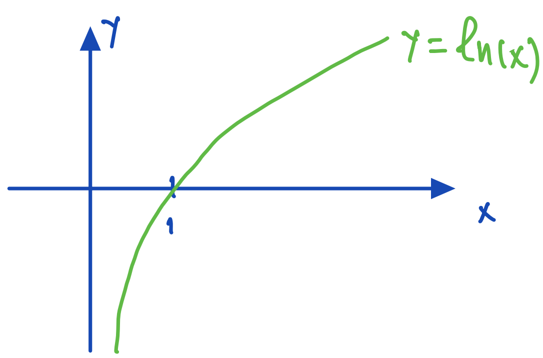
Condition : $\dfrac{2x}{x+1} > 0$

x		-1		0		
$2x$	$-$		$-$	0	$+$	0
$x+1$	$-$		$+$		$+$	-1
$\dfrac{2x}{x+1}$	$+$	‖	$-$	0	$+$	

$$ED(f) =]-\infty; -1[\cup]0; +\infty[$$

② **Aucune parité**, ED(f) pas symétrique par rapport à l'origine.

③ **Signe de f(x)**:



$$\ln\left(\frac{2x}{x+1}\right) = 0 \Leftrightarrow \frac{2x}{x+1} = 1$$

$$\Rightarrow 2x = x+1 \Leftrightarrow x = 1$$

x		-1		0		1	
$f(x)$	$+$	‖	//////	‖	$-$	0	$+$

$f(2) = \ln\left(\frac{4}{3}\right) \cong 0.29$

$f(0.5) = \ln\left(\frac{1}{1.5}\right) \cong -0.41$

$f(-2) = \ln\left(\frac{-4}{-1}\right) \cong 1.39$

④ AV : • $x = -1$ $\qquad \lim\limits_{\substack{x \to -1 \\ <}} \ln\left(\frac{2x}{x+1}\right) \underset{\text{“}\ln\left(\underbrace{\frac{-2}{0^-}}_{+\infty}\right)\text{”}}{=} +\infty$

$\Rightarrow$ AVG : $x = -1$

• $x = 0$: $\qquad \lim\limits_{\substack{x \to 0 \\ >}} \ln\left(\frac{2x}{x+1}\right) \underset{\text{“}\ln\left(\frac{0^+}{1}\right)\text{”}}{=} -\infty$

$\Rightarrow$ AVD : $x = 0$

AH : $\qquad \lim\limits_{x \to \infty} \ln\left(\frac{2x}{x+1}\right) = \lim\limits_{x \to \infty} \ln\left(\frac{\cancel{x} \cdot 2}{\cancel{x}\left(1 + \frac{1}{x}\right)}\right) = \ln(2)$

$\Rightarrow$ $y = \ln(2)$ est une AH

⑤ Croissance.

$(\ln(u))' = \frac{u'}{u}$; $\left(\frac{2x}{x+1}\right)' = \frac{2(x+1) - 2x \cdot 1}{(x+1)^2} = \frac{2}{(x+1)^2}$

$f'(x) = \frac{\cancel{2}}{(x+1)^{\cancel{2}}} \cdot \frac{\cancel{x+1}}{\cancel{2}x} = \frac{1}{x(x+1)}$

x		-1		0	
$f'(x)$	$+$	‖	$-$	‖	$+$
$f(x)$	↗	‖		‖	↗

$f(x)$ est toujours croissante

⑥ Courbure

$$f'(x) = \frac{1}{x^2+x}$$

$$f''(x) = -\frac{2x+1}{(x^2+x)^2}$$

zéros : $-\frac{1}{2}$

x		-1		$-1/2$		0	
$f''(x)$	$+$	‖	$+$	0	$-$	‖	$-$
$f(x)$	convexe	‖				‖	concave