Exercice 1

a) $\int_{-4}^{1} (x^2 - 4x)\,dx = \left[\frac{x^3}{3} - 2x^2\right]_{-4}^{1} = \left(\frac{1}{3} - 2\right) - \left(\frac{-64}{3} - 32\right) = \underline{\frac{155}{3}}$

b) $\int_{1}^{2} \left(x - \frac{1}{x}\right) dx = \left[\frac{x^2}{2} - \ln(|x|)\right]_{1}^{2} = \left(2 - \ln(2)\right) - \left(\frac{1}{2} - 0\right)$

$= \underline{\frac{3}{2} - \ln(2)}$

c) $\int_{1}^{4} \left(\frac{1}{2} x^{-\frac{1}{2}} - \frac{2}{x}\right) dx = \left[\sqrt{x} - 2\ln(|x|)\right]_{1}^{4} = \left(2 - 2\ln(4)\right) - 1$

$= \underline{1 - 2\ln(4)}$

d) $\int_{0}^{\pi} \sin(2x)\,dx = \left[-\frac{1}{2}\cos(2x)\right]_{0}^{\pi} = -\frac{1}{2}\cos(2\pi) + \frac{1}{2}\cos(0)$

$= -\frac{1}{2} + \frac{1}{2} = \underline{0}$

Exercice 2

a) $\int_{0}^{1} e^{1-2x}\,dx = \left[-\frac{1}{2} e^{1-2x}\right]_{0}^{1} = \left(-\frac{1}{2} e^{-1}\right) - \left(-\frac{1}{2} e^{1}\right)$

$= \underline{\frac{1}{2}\left(-\frac{1}{e} + e\right)}$

b) $\int_{0}^{1/2} \frac{3x}{1 - x^2}\,dx = -\frac{3}{2}\left[\ln(|1 - x^2|)\right]_{0}^{1/2} = -\frac{3}{2}\left(\ln\left(\frac{3}{4}\right) - \ln(1)\right)$

$= -\frac{3}{2}\ln\left(\frac{3}{4}\right)$

candidat: $K \cdot \ln(|1 - x^2|)$

(candidat)' : $K \cdot \frac{-2x}{1 - x^2} \Rightarrow -2K = 3 \Rightarrow K = \frac{-3}{2}$

c) $\displaystyle\int_1^2 \frac{x^3}{(x^4+1)^2}\,dx = -\frac{1}{4}\left[\frac{1}{x^4+1}\right]_1^2 = -\frac{1}{4}\left(\frac{1}{17}-\frac{1}{2}\right) = -\frac{1}{4}\left(\frac{-15}{34}\right) = \frac{15}{136}$

candidat : $K\cdot(x^4+1)^{-1}$

(candidat)' : $-K\,(x^4+1)^{-2}\cdot 4x^3 = -4K\,\dfrac{x^3}{(x^4+1)^2} \Rightarrow K = -\frac{1}{4}$

Exercice 3

On cherche les intersections des deux courbes:

$$\cancel{x^2}\cancel{-4} = x^3 + \cancel{x^2} - 4x \cancel{-4}$$

$$x^3 - 4x = 0$$

$$x(x^2-4) = 0$$

$$x(x-2)(x+2) = 0$$

Il y a 3 points d'intersection d'abcisse $-2, 0, 2$; donc il y a 2 zones.

$$A_1 = \left|\int_{-2}^{0}\left[(x^3+x^2-4x-4)-(x^2-4)\right]dx\right| = \left|\int_{-2}^{0}(x^3-4x)\,dx\right|$$

$$= \left|\left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0}\right| = \left|0-(4-8)\right| = 4$$

$$A_2 = \left|\int_{0}^{2}\left[(x^3+x^2-4x-4)-(x^2-4)\right]dx\right| = \left|\int_{0}^{2}(x^3-4x)\,dx\right|$$

$$= \left|\left[\frac{x^4}{4} - 2x^2\right]_{0}^{2}\right| = \left|(4-8)-0\right| = 4$$

L'aire entre les deux courbes: $A_1 + A_2 = 8$