02.11.23

1.3.2 Vérifier les égalités suivantes :

a) $\int \left(-\frac{1}{2x^2}+\frac{x}{2}\right) dx = \frac{1}{2x}+\frac{x^2}{4}+c$, avec $c \in \mathbb{R}$;

$$\int f(x)\,dx = F(x) + c$$

où $F'(x) = f(x)$

$$F(x) = \frac{1}{2x} + \frac{x^2}{4} = \frac{1}{2}\cdot\frac{1}{x} + \frac{1}{4}x^2 = \frac{1}{2}\cdot x^{-1} + \frac{1}{4}x^2$$

$$F'(x) = \frac{1}{2}\cdot(-1)\,x^{-2} + \frac{1}{4}\cdot 2\cdot x^1$$

$$= -\frac{1}{2}\,\frac{1}{x^2} + \frac{1}{2}\,x = f(x)$$

b) $\displaystyle\int \underbrace{\left(4x^2 - \frac{7}{3} - \frac{14}{3x^3}\right)}_{f(x)} dx = \underbrace{\frac{4x^5 - 7x^3 + 7}{3x^2}}_{F(x)} + c$, avec $c \in \mathbb{R}$;

$$F'(x) = \frac{(20x^4 - 21x^2)\cdot 3x^2 - (4x^5 - 7x^3 + 7)\cdot 6x}{9x^4} = \frac{60x^6 - 63x^4 - 24x^6 + 42x^4 - 42x}{9x^4}$$

$u = 4x^5 - 7x^3 + 7$; $u' = 20x^4 - 21x^2$

$v = 3x^2$; $v' = 6x$

$$= \frac{36x^6 - 21x^4 - 42x}{9x^4} = \frac{\overset{4}{\cancel{36}}\,\overset{x^2}{\cancel{x^6}}}{\underset{1}{\cancel{9}}\,\cancel{x^4}} - \frac{\overset{7}{\cancel{21}}\,\cancel{x^4}}{\underset{3}{\cancel{9}}\,\cancel{x^4}} - \frac{\overset{14}{\cancel{42}}\,\cancel{x}}{\underset{3}{\cancel{9}}\,\underset{x^3}{\cancel{x^4}}} = 4x^2 - \frac{7}{3} - \frac{14}{3x^3} = f(x)$$

1.3.3 Calculer :

a) $\int 3\,dx$ $= 3x + C \quad , \; C \in \mathbb{R}$

b) $\int 5x\,dx$ $= \frac{5}{2} x^2 + C$

c) $\int (2x+1)\,dx$ $=$

d) $\int (5x-4)\,dx$ $=$

e) $\int (2x^2-3x+2)\,dx$ $=$

1.3.4 Calculer :

a) $\displaystyle\int \frac{dx}{x^2} = \int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{1}{-2+1}\,x^{-2+1} + c = -x^{-1} + c = -\frac{1}{x} + c$

b) $\displaystyle\int \frac{2dx}{x^3} = \int \frac{2}{x^3}\,dx = 2\int x^{-3}\,dx =$

c) $\displaystyle\int \frac{-7dx}{x^5} = \int \frac{-7}{x^5}\,dx = -7\int x^{-5}\,dx =$

d) $\displaystyle\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx =$

$$\boxed{n \neq -1 \qquad \int x^n\,dx = \frac{1}{n+1}\,x^{n+1} + c}$$

Propriétés des intégrales

① $\int \left(f(x) + g(x) \right) dx = \int f(x)\, dx + \int g(x)\, dx$

② $\int K \cdot f(x)\, dx = K \cdot \int f(x)\, dx \quad , \quad K \in \mathbb{R}$