1.5.1 On donne les vecteurs $\vec{a} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix}$ et $\vec{c} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$.

a) Calculer les produits vectoriels $\underline{\vec{a} \times \vec{b}}$, $\underline{\vec{a} \times \vec{c}}$, $\underline{\vec{b} \times \vec{c}}$, $\underline{\left(\vec{a} + \vec{b}\right) \times \vec{b}}$, $(2\vec{a}) \times \left(-3\vec{b}\right)$, $\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)$, $\left(\vec{a} \times \vec{b}\right) \times \vec{c}$ et $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$.

$$\left(\vec{a} + \vec{b}\right) \times \vec{b} = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} =$$

Pour calculer le produit vectoriel, on utilise un <u>pseudo-déterminant</u> :

$$\begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} = \left|\begin{array}{ccc|cc} \vec{e}_1 & 2 & 0 & \vec{e}_1 & 2 \\ \vec{e}_2 & 4 & 4 & \vec{e}_2 & 4 \\ \vec{e}_3 & 5 & 2 & \vec{e}_3 & 5 \\ - & - & - \; + & + & + \end{array}\right. = \begin{pmatrix} 4 \cdot 2 - 4 \cdot 5 \\ 0 \cdot 5 - 2 \cdot 2 \\ 2 \cdot 4 - 0 \cdot 4 \end{pmatrix} = \begin{pmatrix} -12 \\ -4 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \left|\begin{array}{ccc|cc} \vec{e}_1 & a_1 & b_1 & \vec{e}_1 & a_1 \\ \vec{e}_2 & a_2 & b_2 & \vec{e}_2 & a_2 \\ \vec{e}_3 & a_3 & b_3 & \vec{e}_3 & a_3 \\ - & - & - \; + & + & + \end{array}\right. = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

1.5.1 On donne les vecteurs $\vec{a} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix}$ et $\vec{c} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$.

$$\vec{a} \times \vec{b} = \begin{pmatrix} -12 \\ -4 \\ 8 \end{pmatrix}$$

$$(2\vec{a}) \times \left(-3\vec{b}\right) = -6\left(\vec{a} \times \vec{b}\right)$$

$$\begin{aligned}
\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right) &= \left(\vec{a} + \vec{b}\right) \times \vec{a} + \left(\vec{a} + \vec{b}\right) \times \left(-\vec{b}\right) \\
&= \underbrace{\left(\vec{a} \times \vec{a}\right)} + \left(\vec{b} \times \vec{a}\right) + \left(\vec{a} \times (-\vec{b})\right) + \underbrace{\left(\vec{b} \times (-\vec{b})\right)} \\
&= \vec{0} + \left(\vec{b} \times \vec{a}\right) + \left(\vec{b} \times \vec{a}\right) \quad \vec{0} = -2\left(\vec{a} \times \vec{b}\right) = \begin{pmatrix} 24 \\ 8 \\ -16 \end{pmatrix}
\end{aligned}$$

$$\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \begin{pmatrix} -36 \\ 52 \\ -28 \end{pmatrix}$$

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \begin{pmatrix} 6 \\ 40 \\ -4 \end{pmatrix}$$

Le produit n'est pas associatif

Soit $\mathcal{B} = (\vec{e}_1, \vec{e}_2, \vec{e}_3)$ la base canonique de V_3.

Calculons :

$$\vec{e}_1 \times \vec{e}_2 = \vec{e}_3$$

$$\vec{e}_2 \times \vec{e}_1 = -\vec{e}_3$$

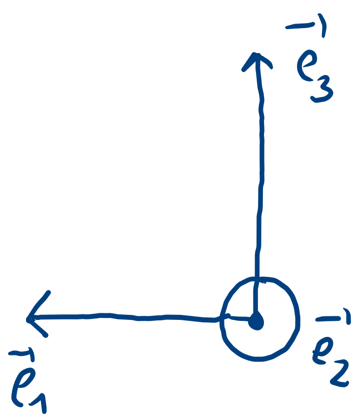
$$\vec{e}_1 \times \vec{e}_3 = -\vec{e}_2$$

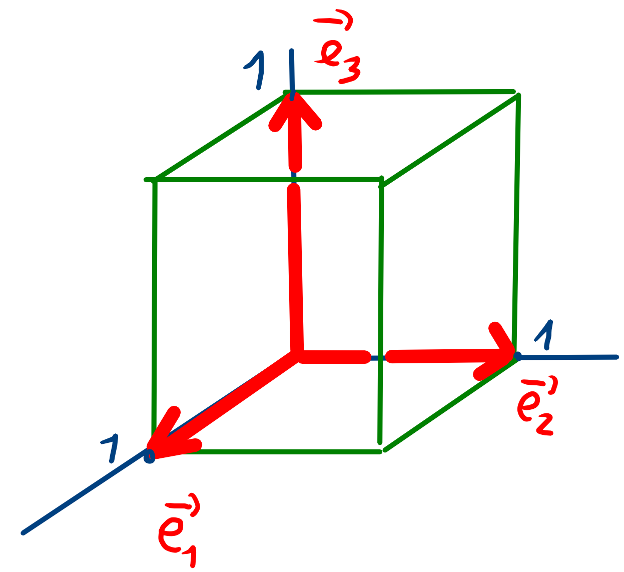
$$\vec{e}_3 \times \vec{e}_1 = \vec{e}_2$$

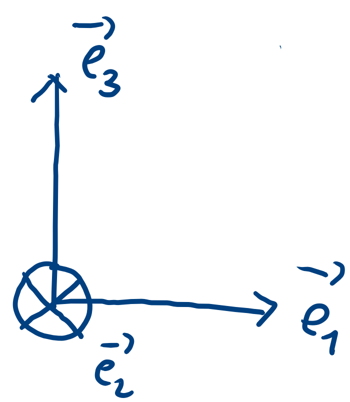
$$\vec{e}_2 \times \vec{e}_3 = \vec{e}_1$$

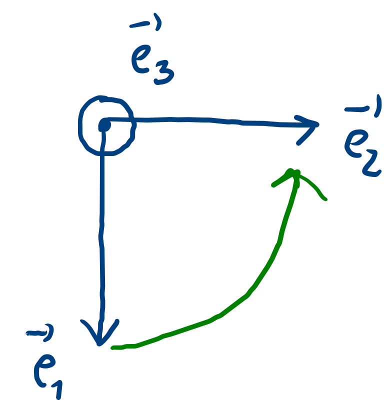
$$\vec{e}_3 \times \vec{e}_2 = -\vec{e}_1$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

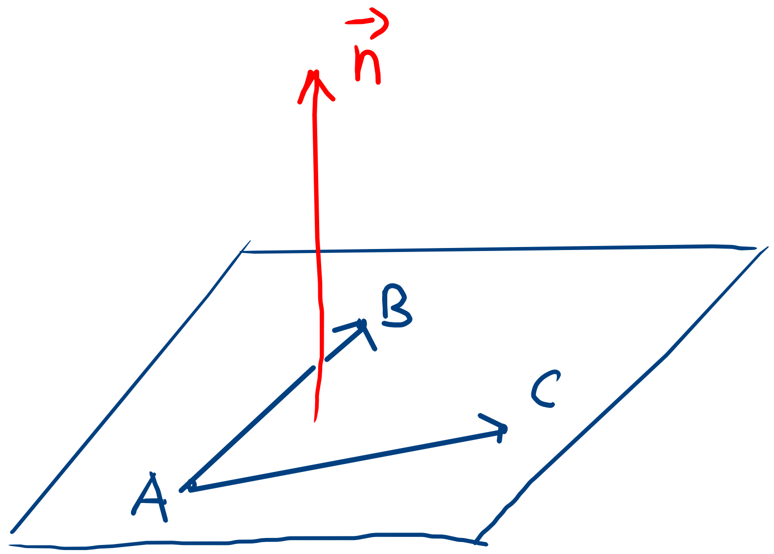








1.5.2 Former un vecteur normal au plan ABC, si $A(0;2;1)$, $B(0;1;0)$ et $C(1;0;2)$.



$$\vec{n} = \vec{AB} \times \vec{AC}$$