Série 1 – Nombres complexes

Exercice 1

Exprimer sous forme algébrique les nombres complexes.

a) $(2+3\mathrm{i})+(-1+6\mathrm{i})$

b) $(5+\mathrm{i})-(3-2\mathrm{i})$

c) $(1+\mathrm{i})(3-2\mathrm{i})$

d) $(4+\mathrm{i})(-5+3\mathrm{i})$

e) $(2-\mathrm{i})^2$

f) $(x+\mathrm{i}y)(x'+\mathrm{i}y')$

g) $(x+\mathrm{i}y)^2$

h) $(2-3\mathrm{i})(2+3\mathrm{i})$

i) $(a+\mathrm{i}b)(a-\mathrm{i}b)$

Exercice 2

Écrire sous forme algébrique les nombres complexes.

a) $\dfrac{1+\mathrm{i}}{3+4\mathrm{i}}$

b) $\dfrac{1}{\sqrt{3}+2\mathrm{i}}$

c) $\dfrac{1+4\mathrm{i}}{2-3\mathrm{i}}+\dfrac{3-\mathrm{i}}{2+3\mathrm{i}}$

d) $(2+3\mathrm{i})\,(5-\mathrm{i})+(4+3\mathrm{i})^2$

e) $\left[3\,;\,\frac{2\pi}{3}\right]$

f) $\left[4\,;\,\frac{7\pi}{4}\right]$

Exercice 3

Soit $z_1=-1+2\mathrm{i}$ et $z_2=1-\mathrm{i}$.
Ecrire sous forme algébrique les nombres complexes.

a) ${z_1}^2-2\,z_2$

b) $z_1\,{z_2}^2$

c) $\dfrac{z_1}{z_2}$

d) $\dfrac{1}{z_1}+\dfrac{1}{z_2}$

e) $\dfrac{1}{{z_1}^2}+\dfrac{1}{{z_2}^2}$

Exercice 4

Écrire sous forme trigonométrique les nombres complexes suivants :

a) $z_1=3$

b) $z_2=-4$

c) $z_3=2\mathrm{i}$

d) $z_4=-1+\mathrm{i}$

e) $z_5=-\sqrt{3}+\mathrm{i}$

f) $z_6=\frac{-17}{5}$

g) $z_7=-6\sqrt{3}+6\mathrm{i}$

h) $z_8=-5\mathrm{i}$

i) $z_9=\sqrt{6}+i\sqrt{2}$

Exercice 5

Ecrire sous forme trigonométrique les nombres complexes suivants :

a) $z_1=\sqrt{3}-\mathrm{i}$

b) $z_2=1-\mathrm{i}$

c) $z_1\cdot z_2$

d) z_1+z_2

e) z_1-z_2

f) $w=\dfrac{z_1}{z_2}$

Exercice 1

a) $(2+3\mathrm{i})+(-1+6\mathrm{i})$ $= 1+9i$

b) $(5+\mathrm{i})-(3-2\mathrm{i})$ $= 2+3i$

c) $(1+\mathrm{i})(3-2\mathrm{i})$ $= 3-2i^2+3i-2i = 5+i$

d) $(4+\mathrm{i})(-5+3\mathrm{i})$ $= -20+3i^2+12i-5i = -23+7i$

e) $(2-\mathrm{i})^2$ $= 4+i^2-4i = 3-4i$

f) $(x+\mathrm{i}y)(x'+\mathrm{i}y')$ $= xx'-yy'+(xy'+x'y)i$

g) $(x+\mathrm{i}y)^2$ $= x^2-y^2+2xyi$

h) $(2-3\mathrm{i})(2+3\mathrm{i})$ $= 4-9i^2 = 13$

i) $(a+\mathrm{i}b)(a-\mathrm{i}b)$ $= a^2+b^2$

Exercice 2

a) $\dfrac{1+\mathrm{i}}{3+4\mathrm{i}} \cdot \dfrac{3-4i}{3-4i} = \dfrac{3+4-i}{9+16} = \dfrac{7-i}{25} = \dfrac{7}{25} - \dfrac{1}{25}i$

b) $\dfrac{1}{\sqrt{3}+2\mathrm{i}} \cdot \dfrac{\sqrt{3}-2i}{\sqrt{3}-2i} = \dfrac{\sqrt{3}-2i}{3+4} = \dfrac{\sqrt{3}}{7} - \dfrac{2}{7}i$

c) $\dfrac{1+4\mathrm{i}}{2-3\mathrm{i}} + \dfrac{3-\mathrm{i}}{2+3\mathrm{i}} = \dfrac{1+4i}{2-3i}\,\dfrac{2+3i}{2+3i} + \dfrac{3-i}{2+3i}\,\dfrac{2-3i}{2-3i}$

$= \dfrac{2-12+11i}{4+9} + \dfrac{6-3-11i}{4+9}$

$= \dfrac{-10+11i}{13} + \dfrac{3-11i}{13} = \dfrac{-7}{13}$

d) $(2+3\mathrm{i})(5-\mathrm{i})+(4+3\mathrm{i})^2$ $= 10+3+13i+16-9+24i$

$= 20+37i$

e) $\left[3;\frac{2\pi}{3}\right] = 3\left(\cos\left(\frac{2\pi}{3}\right)+\sin\left(\frac{2\pi}{3}\right)i\right) = 3\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = -\frac{3}{2}+\frac{3\sqrt{3}}{2}i$

f) $\left[4;\frac{7\pi}{4}\right] = 4\left(\cos\left(\frac{7\pi}{4}\right)+\sin\left(\frac{7\pi}{4}\right)i\right) = 4\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right) = 2\sqrt{2}-2\sqrt{2}i$

Exercice 3

Soit $z_1 = -1 + 2\mathrm{i}$ et $z_2 = 1 - \mathrm{i}$.

a) $z_1{}^2 - 2\,z_2 = (-1+2i)^2 - 2(1-i) = -3-4i-2+2i = -5-2i$

b) $z_1\,z_2{}^2 = (-1+2i)(1-i)^2 = (-1+2i)(-2i) = 2i-4i^2 = 4+2i$

c) $\dfrac{z_1}{z_2} = \dfrac{-1+2i}{1-i}\cdot\dfrac{1+i}{1+i} = \dfrac{-3+i}{2} = -\dfrac{3}{2}+\dfrac{1}{2}i$

d) $\dfrac{1}{z_1}+\dfrac{1}{z_2} = \dfrac{1}{-1+2i}+\dfrac{1}{1-i} = \dfrac{1}{-1+2i}\cdot\dfrac{-1-2i}{-1-2i}+\dfrac{1}{1-i}\cdot\dfrac{1+i}{1+i}$

$= \dfrac{-1-2i}{5}+\dfrac{1+i}{2} = \dfrac{-2-4i}{10}+\dfrac{5+5i}{10} = \dfrac{3}{10}+\dfrac{1}{10}i$

e) $\dfrac{1}{z_1{}^2}+\dfrac{1}{z_2{}^2} = \dfrac{1}{-3-4i}\cdot\dfrac{-3+4i}{-3+4i}+\dfrac{1}{-2i}\cdot\dfrac{2i}{2i} = \dfrac{-3+4i}{25}+\dfrac{2i}{4}$

$z_1{}^2 = (-1+2i)^2 = -3-4i$

$z_2{}^2 = (1-i)^2 = -2i$

$= \dfrac{-12+16i+50i}{100} = \dfrac{-12}{100}+\dfrac{66}{100}i$

$= \dfrac{-3}{25}+\dfrac{33}{50}i$

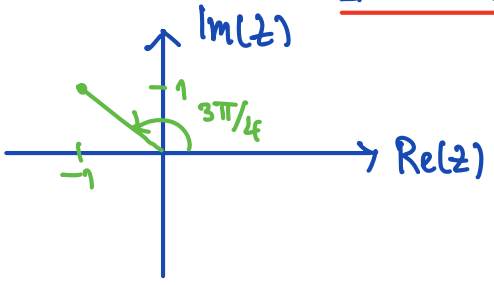
Exercice 4

a) $z_1 = 3$ = $[3;0]$

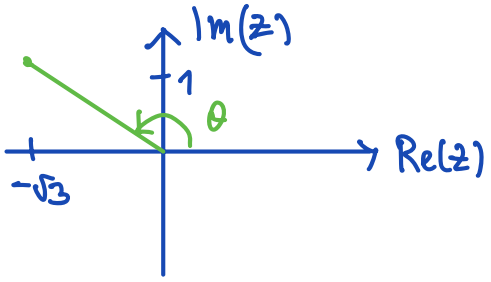
b) $z_2 = -4$ = $[4;\pi]$

c) $z_3 = 2\mathrm{i}$ = $\left[2;\frac{\pi}{2}\right]$

d) $z_4 = -1+\mathrm{i}$ = $\left[\sqrt{2};\frac{3\pi}{4}\right]$



e) $z_5 = -\sqrt{3} + \mathrm{i}$ $\quad |z_5| = 2$

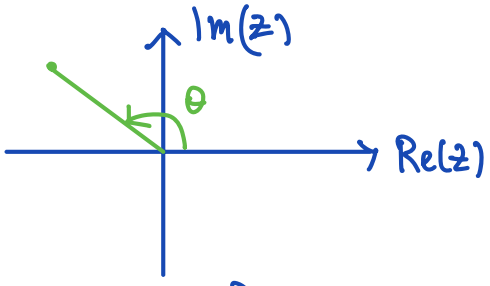


$$\begin{cases} \cos(\theta) = -\frac{\sqrt{3}}{2} \\ \sin(\theta) = \frac{1}{2} \end{cases} \Rightarrow \theta = \frac{5\pi}{6} \quad (150°)$$

$$z_5 = \left[2\,;\, \frac{5\pi}{6}\right]$$

f) $z_6 = \frac{-17}{5} = \left[\frac{17}{5}\,;\,\pi\right]$

g) $z_7 = -6\sqrt{3} + 6\mathrm{i}$ $\quad |z_7| = \sqrt{36 \cdot 3 + 36} = 12$



$$\begin{cases} \cos(\theta) = \frac{-6\sqrt{3}}{12} = -\frac{\sqrt{3}}{2} \\ \sin(\theta) = \frac{6}{12} = \frac{1}{2} \end{cases} \Rightarrow \theta = \frac{5\pi}{6}$$

$$z_7 = \left[12\,;\, \frac{5\pi}{6}\right]$$

h) $z_8 = -5\mathrm{i} = \left[5\,;\,\frac{3\pi}{2}\right]$

i) $z_9 = \sqrt{6} + i\sqrt{2}$ $\quad |z_9| = \sqrt{6+2} = 2\sqrt{2}$

$$\begin{cases} \cos\theta = \frac{\sqrt{6}}{2\sqrt{2}} = \frac{\sqrt{2}\cdot\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}}{2} \\ \sin\theta = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2} \end{cases} \Rightarrow \theta = 30° = \frac{\pi}{6}$$

$$z_9 = \left[2\sqrt{2}\,;\,\frac{\pi}{6}\right]$$

Exercice 5

$\begin{cases} \cos(\theta) = \frac{\sqrt{3}}{2} \\ \sin(\theta) = \frac{-1}{2} \end{cases} \Rightarrow \theta = \frac{11}{6}\pi = 330°$

a) $z_1 = \sqrt{3} - i = \left[2 ; \frac{11\pi}{6}\right]$

b) $z_2 = 1 - i = \left[\sqrt{2} ; \frac{7\pi}{4}\right]$

c) $z_1 \cdot z_2 = \left[2\sqrt{2} ; \frac{19\pi}{12}\right]$

$\arg(z_1 \cdot z_2) = \frac{11\pi}{6} + \frac{7\pi}{4} = \frac{22\pi + 21\pi}{12} = \frac{43\pi}{12} = \frac{19\pi}{12} + 2\pi$

d) $z_1 + z_2 = \sqrt{3} + 1 - 2i$

$|z_1 + z_2| = \sqrt{(\sqrt{3}+1)^2 + 4} = \sqrt{4 + 2\sqrt{3} + 4} = \sqrt{8 + 2\sqrt{3}} \cong 3,38587$

$\begin{cases} \cos(\theta) = \frac{\sqrt{3}+1}{3,38} \\ \sin(\theta) = \frac{-2}{3,38} \end{cases} \Rightarrow \theta = 323,79°$

$\Rightarrow z_1 + z_2 = \left[3,38 ; 323,79°\right]$

e) $z_1 - z_2 = \sqrt{3} - 1 = \left[\sqrt{3} - 1 ; 0\right]$

f) $w = \frac{z_1}{z_2} = \left[\frac{2}{\sqrt{2}} ; \frac{\pi}{12}\right] = \left[\sqrt{2} ; \frac{\pi}{12}\right]$

$\arg\left(\frac{z_1}{z_2}\right) = \frac{11\pi}{6} - \frac{7\pi}{4} = \frac{22\pi}{12} - \frac{21\pi}{12} = \frac{\pi}{12} = 15°$