

Problème 1

$$a) y = (4x^3 - x^2 + 5x - 1)^4$$

$$c) y = \frac{\sqrt{9x-4}}{(3x+8)^3}$$

$$b) y = \frac{(x-5)^3}{(2x+7)^4}$$

$$d) y = (4x+6)^4 (2x-5)^3$$

$$a) y' = 4(4x^3 - x^2 + 5x - 1)(12x^2 - 2x + 5)$$

$$b) u = (x-5)^3 \Rightarrow u' = 3(x-5)^2$$

$$v = (2x+7)^4 \Rightarrow v' = 8(2x+7)^3$$

$$y' = \frac{3(x-5)^2(2x+7)^4 - (x-5)^3 \cdot 8(2x+7)^3}{(2x+7)^8} = \frac{(x-5)^2(2x+7)^3 [3(2x+7) - 8(x-5)]}{(2x+7)^8}$$

$$= \frac{(x-5)^2(-2x+61)}{(2x+7)^5}$$

$$c) y = (9x-4)^{\frac{1}{2}} (3x+8)^{-3}$$

$$(i) y' = \frac{1}{2} \cdot 9(9x-4)^{-\frac{1}{2}} (3x+8)^{-3} + (9x-4)^{\frac{1}{2}} \cdot (-9)(3x+8)^{-4}$$

$$= \frac{\frac{9}{2}}{\sqrt{9x-4} (3x+8)^3} - 9\sqrt{9x-4} \frac{1}{(3x+8)^4} = \frac{9 \left[\frac{1}{2}(3x+8) - (9x-4) \right]}{\sqrt{9x-4} (3x+8)^4}$$

$$= \frac{9}{2} \frac{3x+8-18x+8}{\sqrt{9x-4} (3x+8)^4} = \frac{9}{2} \frac{-15x+16}{\sqrt{9x-4} (3x+8)^4}$$

$$(ii) \text{ ou } u = \sqrt{9x-4} \quad u' = \frac{9}{2\sqrt{9x-4}}$$

$$v = (3x+8)^3 \quad v' = 9(3x+8)^2$$

$$y' = \frac{\frac{9}{2\sqrt{9x-4}} \cdot (3x+8)^3 - \sqrt{9x-4} \cdot 9(3x+8)^2}{(3x+8)^6} = \frac{\frac{9(3x+8)^2}{2\sqrt{9x-4}} - 9\sqrt{9x-4}}{(3x+8)^4} = \dots$$

$$d) y' = 16(4x+6)^3 (2x-5)^3 + (4x+6)^4 \cdot 6(2x-5)^2$$

$$= 2(4x+6)^3 (2x-5)^2 [8(2x-5) + 3(4x+6)]$$

$$= 2(4x+6)^3 (2x-5)^2 (22x-22)$$

$$= 2 \cdot 2^3 (2x+3)^3 (2x-5)^2 \cdot 2(14x-11) = 32(2x+3)^3 (2x-5)^2 (14x-11)$$

Problème 2

$$f(x) = \sqrt{x+3}$$

$$a) \text{ED}(f) = [-3; +\infty[$$

$$b) f'(x) = \frac{1}{2\sqrt{x+3}} \quad \text{ED}(f') =]-3; +\infty[$$

c) La pente de la tangente est égale à $\frac{1}{2}$.

$$\text{On détermine } f'(x) = \frac{1}{2} \Leftrightarrow \frac{1}{2} \cdot \frac{1}{\sqrt{x+3}} = \frac{1}{2}$$

$$\Leftrightarrow \underline{x = -2}$$

Le point sur la courbe: $P(-2; 1)$

$$\text{La tangente: } y = \frac{1}{2}x + h$$

$$\text{Par } P: \quad 1 = -1 + h \Leftrightarrow h = 2$$

$$\Rightarrow \boxed{y = \frac{1}{2}x + 2}$$

Problème 3

$$f'(x) = 4\left(1 - \frac{x}{2}\right)^3 \cdot \left(-\frac{1}{2}\right) = -2\left(1 - \frac{x}{2}\right)^3 = 2\left(\frac{x}{2} - 1\right)^3$$

$$a) f'(4) = 2(2-1)^3 = 2$$

$$y = 2x + h, \quad \text{par } P_0(4; 1) \Rightarrow \underline{y = 2x - 7}$$

$$b) f'(3) = 2\left(\frac{3}{2} - 1\right)^3 = 2 \cdot \left(\frac{1}{2}\right)^3 = \frac{1}{4}$$

$$y = \frac{1}{4}x + h, \quad \text{par } P_1\left(3; \frac{1}{16}\right) \Rightarrow \underline{y = \frac{1}{4}x - \frac{11}{16}}$$

$$\frac{1}{16} = \frac{3}{4} + h \Leftrightarrow h = \frac{1}{16} - \frac{3}{4} = \frac{1-12}{16} = \frac{-11}{16}$$