04.03.26

g) $f(x) = \sqrt{1 - x^2}$

Recherche de ED :

x		-1		1	
$1-x^2$	$-$	0	$+$	0	$-$

$ED(f) = [-1 ; 1]$

- $f'(x) = \dfrac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}} = \dfrac{-x}{\sqrt{1-x^2}}$

$\left(\sqrt{u}\right)' = \dfrac{u'}{2\sqrt{u}}$

$\left[= -x \cdot (1-x^2)^{-\frac{1}{2}} \right]$ variante

- $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$u = -x \quad ; \quad u' = -1$

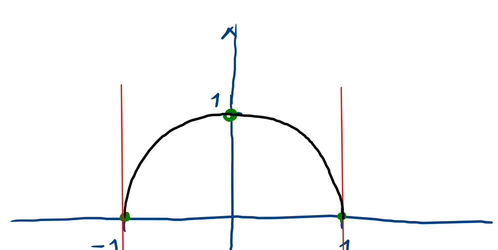
$v = \sqrt{1-x^2} \quad ; \quad v' = \dfrac{-x}{\sqrt{1-x^2}}$

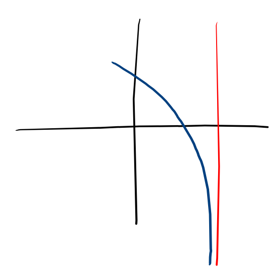
$$f''(x) = \frac{-\sqrt{1-x^2} + x \cdot \dfrac{-x}{\sqrt{1-x^2}}}{1-x^2} = \frac{\dfrac{-(1-x^2) - x^2}{\sqrt{1-x^2}}}{1-x^2}$$

$$= \frac{-1}{(1-x^2)\sqrt{1-x^2}}$$

$ED(f'') =]-1 ; 1[$

x		-1		1	
$f''(x)$		$\Vert$	$-$	$\Vert$	
$f(x)$		0		0	





h) $f(x) = \cos^2(x) = (\cos(x))^2$, sur $[0; 2\pi]$ $\qquad ED(f) = \mathbb{R}$

$$f'(x) = 2\cos(x) \cdot (-\sin(x)) = -2\sin(x)\cos(x)$$

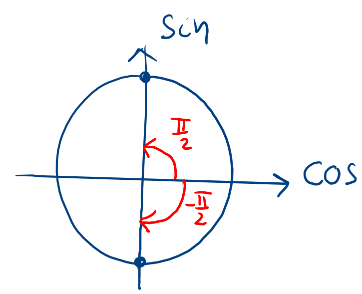
CRM : $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$

$$= -\sin(2x)$$

$$f''(x) = -\cos(2x) \cdot 2 = -2\cos(2x)$$

Cherchons les zéros de $f''(x)$:

$$-2 \cdot \cos(2x) = 0$$
$$\cos(2x) = 0$$



$$\left[\begin{array}{l} 2x = \frac{\pi}{2} + k \cdot 2\pi \\ 2x = -\frac{\pi}{2} + k \cdot 2\pi \end{array}\right. \iff \left[\begin{array}{l} x = \frac{\pi}{4} + k\pi \\ x = -\frac{\pi}{4} + k\pi \end{array}\right.$$

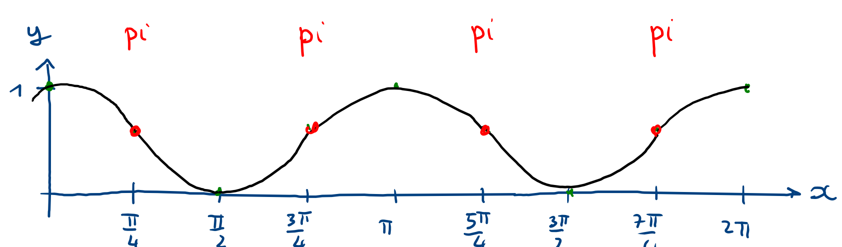
Dans l'intervalle $[0; 2\pi]$: $\frac{\pi}{4}$, $\frac{5\pi}{4}$, $\frac{3\pi}{4}$, $\frac{7\pi}{4}$

45° 135°

x	0		$\frac{\pi}{4}$		$\frac{3\pi}{4}$		$\frac{5\pi}{4}$		$\frac{7\pi}{4}$		2π
$f''(x)$		$-$	0	$+$	0	$-$	0	$+$	0	$-$	
$f(x)$		$\cap$	$\boxed{\text{pi}}$	$\cup$	$\boxed{\text{pi}}$	$\cap$	$\boxed{\text{pi}}$	$\cup$	$\boxed{\text{pi}}$	$\cap$	

$f''(x) = -2\cos(2x)$

$f(x) = \cos^2(x)$

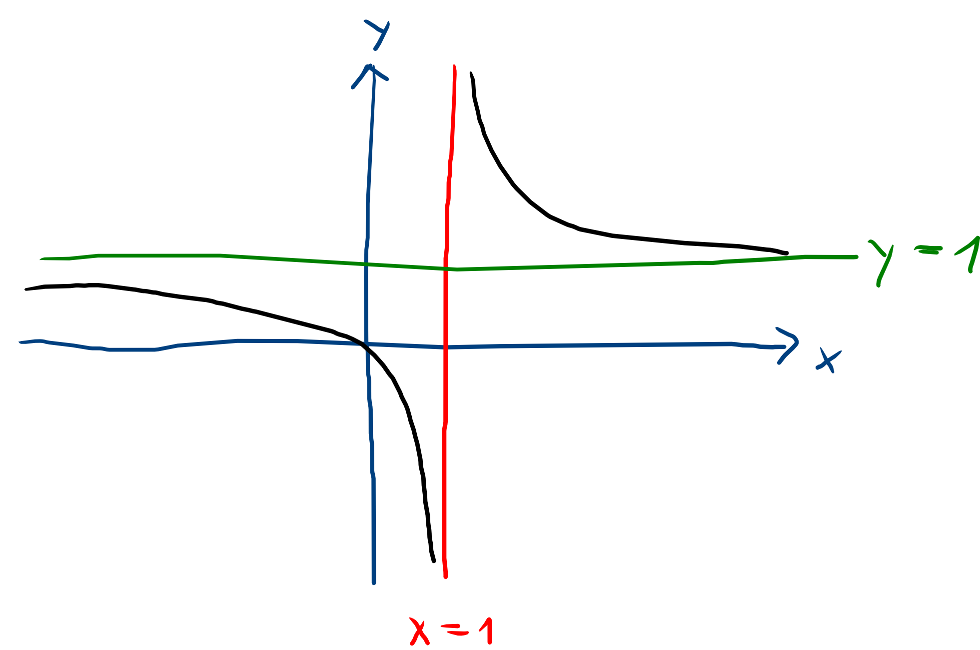


e) $f(x) = \dfrac{x}{x-1}$ $\qquad ED(f) = \mathbb{R} - \{1\}$

$$f'(x) = \frac{1 \cdot (x-1) - x \cdot 1}{(x-1)^2} = \frac{-1}{(x-1)^2} \qquad ED(f') = \mathbb{R} - \{1\}$$

$$f''(x) = \frac{2(x-1)}{(x-1)^4} = \frac{2}{(x-1)^3} \qquad ED(f'') = \mathbb{R} - \{1\} \qquad \left(\frac{1}{u}\right)' = \frac{-u'}{u^2}$$

x		1	
$f''(x)$	$-$	‖	$+$
$f(x)$		‖	



$$\left[(5x-1)^3\right]' = 3(5x-1)^2 \cdot 5$$

$$\left(\sin(2x)\right)' = \cos(2x) \cdot 2$$