07.01.26

f) $f(x) = \dfrac{3x - \sqrt{9x^2 + 2x + 1}}{2x}$ $\Delta < 0$ toujours positif

1) $ED(f) = \mathbb{R}^*$

2) <u>Signe de f :</u>

zéro : $3x = \sqrt{9x^2 + 2x + 1}$ $\quad ()^2$ ⚠

$$\underline{9x^2} = \underline{9x^2} + 2x + 1$$

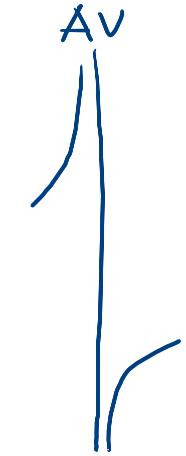
$$2x = -1$$

$$x = -0{,}5$$

$f(-0{,}5) = \dfrac{-1{,}5 - 1{,}5}{-1} \neq 0$ pas de zéro

x		0	
$f(x)$	+	||	−

3) AV : $\lim\limits_{x\to 0} f(x) \underset{"\frac{-1}{0}"}{=} \infty \Rightarrow \begin{cases} \lim\limits_{x \underset{<}{\to} 0} f(x) = +\infty \\ \lim\limits_{x\underset{>}{\to}0} f(x) = -\infty \end{cases}$



$x = 0$

4) AH/AO : a) à gauche

$$m = \lim_{x\to-\infty} \frac{3x - \sqrt{9x^2+2x+1}}{2x^2} = \lim_{x\to-\infty} \frac{3x + 3x\sqrt{1+\dots}}{2x^2}$$

$$= \lim_{x\to-\infty} \frac{3\cancel{x}\left(1+\sqrt{1+\dots}\right)}{2\cancel{x^2}x} = 0$$

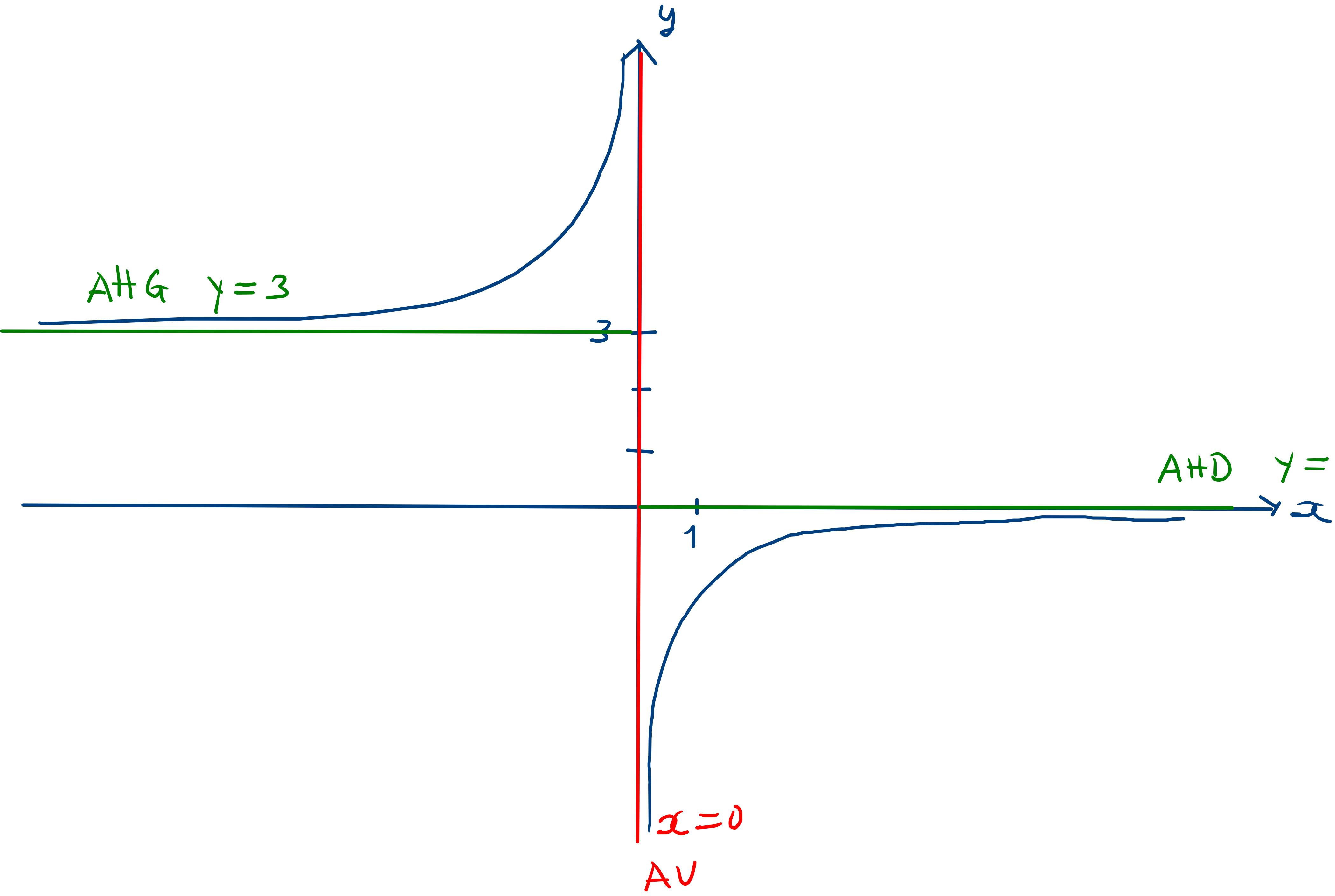
$$\lim_{x\to-\infty} \frac{3x - \sqrt{9x^2(1+\dots)}}{2x} = \lim_{x\to-\infty} \frac{3x + 3x\sqrt{1+\dots}}{2x}$$

$$= \lim_{x\to-\infty} \frac{3\cancel{x}\left(1+\sqrt{1+\dots}\right)}{2\cancel{x}} = 3 \Rightarrow \boxed{AHG : y = 3}$$

A droite :

$$\lim_{x \to +\infty} \frac{3x - \sqrt{9x^2 + 2x + 1}}{2x} = \lim_{x \to +\infty} \frac{3x - 3x\sqrt{1 + \frac{2}{9x} + \frac{1}{9x^2}}}{2x}$$

$$= \lim_{x \to +\infty} \frac{3\cancel{x}\left(1 - \sqrt{1 + \ldots}\right)}{2\cancel{x}} = 0 \qquad \Rightarrow \boxed{\text{AHD} \quad y = 0}$$



y
AHG y = 3
3
AHD y = 0
x
1
x = 0
AV