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1.3.3 Résoudre dans $\mathbb{C}$ les équations ci-dessous :

a) $z^4 - (6+3i)z^3 + (8+12i)z^2 = 0$

$$z^2\left(z^2 - (6+3i)z + (8+12i)\right) = 0$$

① $z = 0$

② $z^2 - (6+3i)z + (8+12i) = 0$

$a = 1$; $b = -(6+3i)$; $c = 8+12i$

$\Delta = -5-12i = (6+3i)^2 - 4(8+12i) = 36 - 9 + 36i - 32 - 48i$

Les racines de Δ : $-2+3i$ et $2-3i$, $\pm(2-3i)$

$$\begin{cases} a^2 - b^2 = -5 \\ a^2 + b^2 = 13 \\ 2ab = -12 \end{cases}$$ "$\sqrt{\Delta}$"

w tel que $w^2 = -5-12i$

$w = a+bi$

$$z = \frac{6+3i \pm (2-3i)}{2}$$

$$z = \begin{cases} \dfrac{6+3i+2-3i}{2} = 4 \\[2ex] \dfrac{6+3i-2+3i}{2} = \dfrac{4+6i}{2} = 2+3i \end{cases}$$

$S = \{0\ ;\ 4\ ;\ 2+3i\}$

1.3.2

e) $\underbrace{z^3 - 6z^2 + 13z - 10}_{p} = 0$

① $p(1) = \times$

$p(-1) = \times$

$p(2) = 0 \Rightarrow (z-2)$ divise p

$p(-2) = \times$

On divise p par $z-2$ (Horner) :

	1	−6	13	−10
2		2	−8	10
	1	−4	5	0

② $p = (z-2)(z^2 - 4z + 5)$ factorisation dans $\mathbb{R}[z]$

$\Delta = 16 - 20 = -4 = 4i^2 \Rightarrow \sqrt{\Delta} = \pm 2i$

$z = \dfrac{4 \pm 2i}{2} = \begin{cases} 2+i \\ 2-i \end{cases}$ racines conjuguées

③ $p = (z-2)\big(z-(2+i)\big)\big(z-(2-i)\big)$ dans $\mathbb{C}[z]$

Les solutions de $p = 0$: $S = \{2 ; 2+i ; 2-i\}$