12.11.25

2.5.11 Calculer :

a) $\lim\limits_{n\to\infty} \dfrac{2n^2-3}{4n} = \infty$

b) $\lim\limits_{n\to\infty} \left(n \cdot \dfrac{1}{1-n}\right)$

c) $\lim\limits_{n\to\infty} \dfrac{3^n - 3^{n-1}}{2+3^n}$

d) $\lim\limits_{n\to\infty} (\sqrt{n^2+n} - n)$

e) $\lim\limits_{n\to\infty} \left(n^2 + \dfrac{2n+n^2-n^3}{n-1}\right)$

f) $\lim\limits_{n\to\infty} \dfrac{1+2+\ldots+n}{n^2}$

Ici, le but est de calculer, si elle existe, la limite de ces suites

a) $$\lim_{n\to\infty} \frac{2n^2-3}{4n} = \lim_{n\to\infty} \frac{n^{\cancel{2}}\left(2-\frac{3}{n^2}\right)}{\cancel{n}\cdot 4} = \lim_{n\to\infty} \frac{n\left(2-\frac{3}{n^2}\right)}{4} \underset{\text{“}\frac{\infty}{4}\text{”}}{=} \infty$$

La suite diverge.

b) $$\lim_{n\to\infty} \left(n \cdot \frac{1}{1-n}\right) \underset{\text{“}\infty\cdot 0\text{”}}{\overset{\text{Ind}}{=}} \lim_{n\to\infty} \frac{n}{1-n} \underset{\text{“}\frac{\infty}{-\infty}\text{”}}{\overset{\text{Ind}}{=}} \lim_{n\to\infty} \frac{\cancel{n}^{\,1}}{\cancel{n}\left(\frac{1}{n}-1\right)} = \frac{1}{-1} = -1$$

“$\infty \cdot 0$” est une forme indéterminée

c) $$\lim_{n\to\infty} \frac{3^n - 3^{n-1}}{3^n + 2} \underset{\text{“}\frac{\infty}{\infty}\text{”}}{\overset{\text{Ind}}{=}} \lim_{n\to\infty} \frac{\cancel{3^n}\overbrace{\left(1-3^{-1}\right)}^{1-\frac{1}{3}=\frac{2}{3}}}{\cancel{3^n}\left(1+\frac{2}{3^n}\right)} =$$

$$= \lim_{n\to\infty} \frac{\frac{2}{3}}{1+\frac{2}{3^n}} = \frac{\frac{2}{3}}{1} = \frac{2}{3}$$

d) $\lim_{n\to\infty} (\sqrt{n^2+n} - n) \overset{\text{Ind}}{\underset{\text{"}\infty-\infty\text{"}}{=}} \lim_{n\to\infty} \left(\sqrt{n^2(1+\frac{1}{n})} - n\right) = \lim_{n\to\infty} \left(n\sqrt{1+\frac{1}{n}} - n\right)$

$$= \lim_{n\to\infty} \Big[\underbrace{n}_{\infty} \cdot \underbrace{\left(\sqrt{1+\tfrac{1}{n}} - 1\right)}_{0}\Big] \overset{\text{Ind}}{=} \lim_{n\to\infty} \frac{\sqrt{n^2+n} - n}{1} \cdot \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n}$$

$$= \lim_{n\to\infty} \frac{n^2+n-n^2}{n\sqrt{1+\frac{1}{n}} + n} = \lim_{n\to\infty} \frac{\cancel{n}\ 1}{\cancel{n}\left(\sqrt{1+\frac{1}{n}} + 1\right)} = \frac{1}{2}$$

e) $\lim_{n\to\infty} \left(n^2 + \frac{2n+n^2-n^3}{n-1}\right) = \lim_{n\to\infty} \left(n^2 + \frac{-n^3+n^2+2n}{n-1}\right) \overset{\text{Ind}}{\underset{\text{"}\infty-\infty\text{"}}{=}}$

$$= \lim_{n\to\infty} \frac{n^2(n-1) - n^3 + n^2 + 2n}{n-1} = \lim_{n\to\infty} \frac{\cancel{n^3} \cancel{-n^2} \cancel{-n^3} \cancel{+n^2} + 2n}{n-1}$$

$$= \lim_{n\to\infty} \frac{2n}{n-1} = \lim_{n\to\infty} \frac{2\cancel{n}}{\cancel{n}(1-\frac{1}{n})} = 2$$

f) $\lim_{n\to\infty} \frac{1+2+\ldots+n}{n^2} = \lim_{n\to\infty} \frac{n^2+n}{2n^2} = \frac{1}{2}$

$$S = \boxed{n} + \boxed{n-1} + n-2 + \ldots + \boxed{1}$$
$$S = \boxed{1} + \boxed{2} + 3 + \ldots + \boxed{n}$$
$$2S = (n+1) + (n+1) + \ldots\ldots + (n+1)$$

$$2S = n\cdot(n+1) \Rightarrow S = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$$

$$1+2+\ldots+1000 = \frac{1000\cdot 1001}{2} = 500'500$$

Suite arithmétique, suite géométrique