19.09.25

4.1.8 Simplifier les expressions suivantes :

a) $\sqrt{24}$ b) $\sqrt{18}$ c) $\sqrt{243}$ d) $\sqrt{50}$ e) $\sqrt{300}$ f) $\sqrt{54}$

g) $\sqrt{125}$ h) $\sqrt{147}$ i) $\sqrt{80}$ j) $\sqrt{1'000}$ k) $\sqrt{250}$ l) $\sqrt{7'000}$

m) $3\sqrt{5}-4\sqrt{20}+5\sqrt{45}-3\sqrt{80}$ n) $2\sqrt{40}-2\sqrt{90}+\sqrt{4'000}-5\sqrt{10}$

n	n^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

a) $\sqrt{24} = 2\sqrt{6}$; $\sqrt{18} = 3\sqrt{2}$; $\sqrt{243} = 3\sqrt{27} = 9\sqrt{3}$

n) $2\sqrt{40} - 2\sqrt{90} + \sqrt{4000} - 5\sqrt{10}$

$= 4\sqrt{10} - 6\sqrt{10} + 20\sqrt{10} - 5\sqrt{10} = 13\sqrt{10}$

4.1.9 Effectuer et réduire :

a) $(9\sqrt{12}+3)(\sqrt{3}+8)$ b) $(4\sqrt{3}+\sqrt{45})(\sqrt{5}-2\sqrt{27})$

c) $\underbrace{\sqrt{3-2\sqrt{2}}}_{x} \cdot \underbrace{\sqrt{3+2\sqrt{2}}}_{\frac{1}{x}}$ d) $(\sqrt{3}+1)^4$

b) $(4\sqrt{3}+3\sqrt{5})(\sqrt{5}-6\sqrt{3})$

$= 4\sqrt{3}\sqrt{5} - 24\cdot 3 + 3\cdot 5 - 18\sqrt{5}\sqrt{3}$

$= 4\sqrt{15} - 57 - 18\sqrt{15} = -14\sqrt{15} - 57$

c) $\sqrt{(3-2\sqrt{2})(3+2\sqrt{2})} = \sqrt{9-8} = 1$

d) $(\sqrt{3}+1)^2 = 4+2\sqrt{3}$

$(4+2\sqrt{3})^2 = 16+12+24\sqrt{3} = 28+24\sqrt{3}$

4.1.10 Simplifier les expressions suivantes :

a) $\sqrt[3]{\sqrt{7}}$ b) $\sqrt[3]{2^{18}\cdot 5^{12}\cdot 3^{3}}$ c) $\sqrt[4]{64}\cdot\sqrt[4]{4}$ d) $\sqrt[5]{3^{15}}$ e) $(\sqrt[8]{\sqrt[4]{\sqrt{2}}})^{128}$

f) $\sqrt{3\sqrt{3}}$ g) $\sqrt[3]{5\sqrt{5\sqrt{5}}}$ h) $\sqrt{2\sqrt[3]{2}}$ i) $\sqrt[3]{3\sqrt[3]{3^4\sqrt[3]{3^6}}}$ j) $\sqrt[3]{2\sqrt[6]{\frac{2^{14}}{\sqrt[3]{2^6}}}}$

j) $\left(2\left(\frac{2^{14}}{2^2}\right)^{\frac{1}{6}}\right)^{\frac{1}{3}} = \left(2\cdot\left(2^{12}\right)^{\frac{1}{6}}\right)^{\frac{1}{3}}$ $\boxed{2^{\frac{6}{3}} = 2^2}$

$= \left(2\cdot 2^{2}\right)^{\frac{1}{3}} = \left(2^{3}\right)^{\frac{1}{3}} = 2$

f) $\sqrt{3\sqrt{3}} = \left(3\, 3^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(3^{\frac{3}{2}}\right)^{\frac{1}{2}} = 3^{\frac{3}{4}} = \sqrt[4]{3^3} = \sqrt[4]{27}$

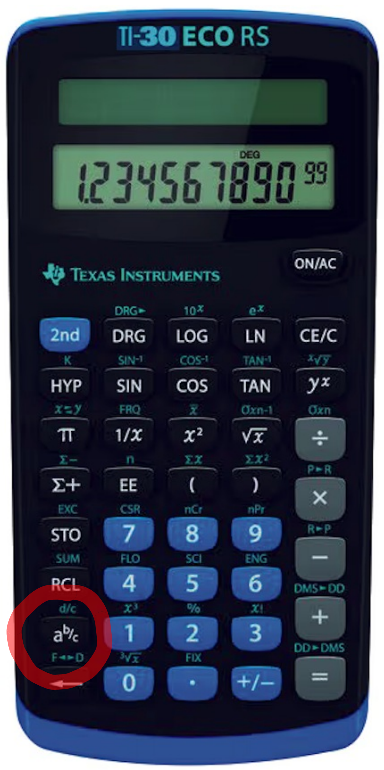
4.1.11 Simplifier les expressions suivantes :

a) $\sqrt[5]{a^3} \cdot (\sqrt[5]{a})^2$ b) $\sqrt[3]{a} \cdot (\sqrt[3]{a})^2$ c) $\sqrt[5]{a^3} \cdot (\sqrt[5]{a^2})^6$ d) $\sqrt[4]{a^3} \cdot \sqrt[3]{a^4}$

e) $\sqrt{a} \cdot \sqrt[5]{a^3} \cdot (\sqrt[10]{a})^4$ f) $\sqrt[3]{a} \cdot \sqrt[4]{a^3} \cdot \sqrt[6]{a}$ g) $\sqrt{\sqrt[3]{a}}$ h) $(\sqrt[10]{\sqrt[5]{a}})^{15}$

i) $\dfrac{\sqrt[3]{a^4}}{\sqrt{a}}$ j) $\dfrac{\sqrt[6]{a^5}}{\sqrt[4]{a^3}}$ k) $\dfrac{\sqrt{a} \cdot \sqrt[3]{a}}{\sqrt[4]{a^3}}$ l) $\dfrac{a^3}{\sqrt[3]{a^5} \cdot \sqrt[6]{a}}$

f) $\sqrt[3]{a} \cdot \sqrt[4]{a^3} \cdot \sqrt[6]{a} = a^{\frac{1}{3}} \cdot a^{\frac{3}{4}} \cdot a^{\frac{1}{6}} = a^{\frac{1}{3}+\frac{3}{4}+\frac{1}{6}} = a^{5/4} = \sqrt[4]{a^5}$



$\boxed{1}\ \boxed{a^{b}/_{c}}\ \boxed{3}\ \boxed{+}\ \boxed{3}\ \boxed{a^{b}/_{c}}\ \boxed{4}\ \boxed{+}\ \boxed{1}\ \boxed{a^{b}/_{c}}\ \boxed{6}\ \boxed{=}$

1_1⌟4 $\boxed{\text{2nd}}\ \boxed{d/c}$ 5⌟4

k) $\dfrac{\sqrt{a} \cdot \sqrt[3]{a}}{\sqrt[4]{a^3}} = a^{\frac{1}{2}} \cdot a^{\frac{1}{3}} \cdot a^{-\frac{3}{4}} = a^{\frac{1}{2}+\frac{1}{3}-\frac{3}{4}} = a^{\frac{6+4-9}{12}} = a^{\frac{1}{12}} = \sqrt[12]{a}$

4.1.12 Rendre rationnel les dénominateurs et simplifier les expressions :

a) $\sqrt{\frac{1}{2}}$ b) $\frac{2}{\sqrt[4]{5}}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{1}{2+\sqrt{3}}$ e) $\frac{2}{\sqrt{5}+\sqrt{3}}$ f) $\frac{1}{\sqrt[3]{3}-\sqrt[3]{2}}$

a) $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

b) $\frac{2}{\sqrt[4]{5^1}} \cdot \frac{\sqrt[4]{5^3}}{\sqrt[4]{5^3}} = \frac{2\sqrt[4]{5^3}}{\sqrt[4]{5^4}} = \frac{2\sqrt[4]{5^3}}{5}$

$\sqrt[4]{5} \cdot \sqrt[4]{5^3} = \sqrt[4]{5 \cdot 5^3} = \sqrt[4]{5^4} = 5$

d) $\frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

On amplifie par le conjugé

$\left[\frac{1}{2+\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{3}+3} \right.$ moche ☹ $\left. \right]$

f) $\frac{1}{\sqrt[3]{3}-\sqrt[3]{2}} = \frac{1}{3^{1/3}-2^{1/3}} \cdot \frac{3^{2/3}+2^{1/3}\cdot 3^{1/3}+2^{2/3}}{3^{2/3}+2^{1/3}\cdot 3^{1/3}+2^{2/3}} = \frac{\sqrt[3]{3^2}+\sqrt[3]{6}+\sqrt[3]{2^2}}{1} = \sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}$

$\sqrt[3]{3}-\sqrt[3]{2} = \left(3^{\frac{1}{3}} - 2^{\frac{1}{3}}\right)\left(3^{\frac{2}{3}} + 2^{\frac{2}{3}}\right) = 3 + \underbrace{3^{\frac{1}{3}} \cdot 2^{\frac{2}{3}} - 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}}}_{\neq 0} - 2$

$\sqrt[3]{3}-\sqrt[3]{2} = \left(3^{\frac{1}{3}} - 2^{\frac{1}{3}}\right)\left(3^{\frac{2}{3}} + 2^{\frac{1}{3}} \cdot 3^{\frac{1}{3}} + 2^{\frac{2}{3}}\right) = 1$

$= 3 + 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}} + 3^{\frac{1}{3}}\, 2^{\frac{2}{3}} - 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}} - 2^{\frac{2}{3}}\, 3^{\frac{1}{3}} - 2$

4.2 Exponentielles et logarithmes

4.2.1 Résoudre les équations ci-dessous :

a) $7^{x+6} = 7^{3x+4}$

g) $27^{x-1} = 9^{2x-3}$

Soit $a \in \mathbb{R}_+^* - \{1\}$

$$\boxed{a^x = a^y \iff x = y}$$

a) $x+6 = 3x+4$

$2x = 2$

$x = 1$

d) $9^{(x^2)} = 3^{3x+2}$

$$\boxed{\begin{array}{l} (x^m)^n = x^{m \cdot n} \\ \frac{1}{x^1} = x^{-1} \end{array}}$$

$(3^2)^{x^2} = 3^{3x+2}$

$3^{2x^2} = 3^{3x+2} \Rightarrow 2x^2 = 3x+2$

...

e) $2^{-100x} = 0,5^{x-4}$

$$\boxed{0,5 = \frac{1}{2} = 2^{-1}}$$

$2^{-100x} = \left(\frac{1}{2}\right)^{x-4}$

$$\boxed{a^x \cdot \underbrace{a^{-x}}_{\frac{1}{a^x}} = a^0 = 1}$$

$2^{-100x} = (2^{-1})^{x-4}$

$2^{-100x} = 2^{-x+4} \Rightarrow -100x = -x+4$

...

h) $2^x \cdot 4^x = -5$

$2^x \cdot 2^{2x} = -5$

$\underbrace{2^{3x}}_{>0} = -5 \Rightarrow S = \emptyset$

k) $3^{4x+2} - 36 \cdot 3^{2x+1} = -243$

$$\left(3^{2x+1}\right)^2 - 36 \cdot 3^{2x+1} + 243 = 0$$

① Changement de variable : $3^{2x+1} = y > 0$

$$y^2 - 36y + 243 = 0$$

$y = 9$ ou $y = 27$

② $y = 9 \Rightarrow 3^{2x+1} = 9 \Rightarrow 3^{2x+1} = 3^2 \Rightarrow 2x+1 = 2$

$\Rightarrow x = \frac{1}{2}$

$y = 27 \Rightarrow 3^{2x+1} = 27 \Rightarrow 3^{2x+1} = 3^3 \Rightarrow 2x+1 = 3$

$x = 1$

$$S = \left\{ \frac{1}{2} ; 1 \right\}$$

1) $5^{1} \cdot 5^{4x-7} - 120 \cdot 5^{2x-3} = 625$

$$5^{4x-6} - 120 \; 5^{2x-3} - 625 = 0$$

posons $y = 5^{2x-3}$

$\Rightarrow$ L'équation devient :

$$y^2 - 120y - 625 = 0$$

1. $10^x = 1000$

   $x = 3$

2. $10^x = 2000 \Rightarrow \boxed{\log}$ 10

   $x \cong 3{,}30$ $\boxed{\ln}$ $e = 2{,}71\ldots$ nombre d'Euler

3. $7^x = 1000$