20.11.25

① $$\lim_{x \to 0} \frac{\cos(x)}{x} = \infty$$

"$\frac{1}{0}$"

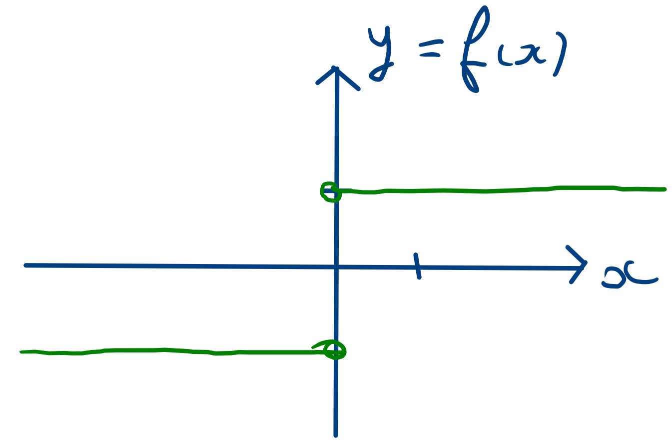
$$\lim_{\substack{x \to 0 \\ <}} \frac{\cos(x)}{x} = -\infty \quad \text{et} \quad \lim_{\substack{x \to 0 \\ >}} \frac{\cos(x)}{x} = +\infty$$

② $$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

2.6.5 Calculer, si elles existent, la limite à gauche, la limite à droite et la limite des fonctions suivantes pour x tendant vers x_0 :

a) $f(x) = \dfrac{|x|}{x} \qquad x_0 = 0$

b) $f(x) = \dfrac{x^2 + |x|}{|x|} \qquad x_0 = 0$



a) $ED(f) = \mathbb{R} - \{0\}$

$$\lim_{\substack{x \to 0 \\ >}} \frac{|x|}{x} = \lim_{\substack{x \to 0 \\ >}} \frac{x}{x} = 1$$

$$\lim_{\substack{x \to 0 \\ <}} \frac{|x|}{x} = \lim_{\substack{x \to 0 \\ <}} \frac{-x}{x} = -1$$

$\lim\limits_{x \to 0} \dfrac{|x|}{x}$ n'existe

$|-7| = 7$

$|-0{,}00001| = 0{,}00001$

Injectif

$f : A \longrightarrow B$

$a, b \in A$ si $f(a) = f(b) \Longleftrightarrow a = b$

- $f : \mathbb{R} \to \mathbb{R}_+$

  $x \longmapsto x^2$

  $1 \longmapsto 1$

  $-1 \longmapsto 1$

- $f : \mathbb{R} \to \mathbb{R}$

  $x \longmapsto x^3 - x$

  $0 \longmapsto 0$

  $1 \longmapsto 0$

$f : \mathbb{R} \to \mathbb{R}$

$x \longmapsto 4x - 7$

$a, b : f(a) = f(b)$

$4a - 7 = 4b - 7$

$4a = 4b$

$a = b$

$f : \mathbb{R}_+ \longrightarrow \mathbb{R}$

$x \longmapsto x^2$

Surjectif

$f : A \longrightarrow B$

$\forall b \in B, \exists a \in A$ tq $f(a) = b$

$f : \mathbb{R} \longrightarrow \mathbb{R}$

$x \longmapsto x^2$

n'existe pas $\longmapsto -1$

$f : \mathbb{R} \longrightarrow \mathbb{R}$

$x \longmapsto x^2 - 100$

n'existe pas $\longmapsto -101$

$f : \mathbb{R} \longrightarrow \mathbb{R}_+$

$x \longmapsto x^2$

$f^r : \mathbb{R}_+ \longrightarrow \mathbb{R}$

$y \longmapsto \sqrt{y}$

x	$f(x)$
0	-100
1	-99
-1	-99

$f : \mathbb{R} \longrightarrow \mathbb{R}_+$

$x \longmapsto x^2$

$x - 1$

h) $$f_8 : \mathbb{R} - \{1\} \to \mathbb{R} - \{1\}, \quad x \mapsto \frac{x+1}{x-1}$$

$$f_8^{r} : \mathbb{R} - \{1\} \to \mathbb{R} - \{1\}$$
$$x \mapsto \frac{-x-1}{1-x}$$

$$\frac{x+1}{x-1} = y \qquad \Rightarrow \qquad x = \ldots$$

$$(x+1) = y(x-1)$$

$$x + 1 = yx - y$$

$$x - yx = -y - 1$$

$$x(1-y) = -y - 1 \qquad \Rightarrow \qquad x = \frac{-y-1}{1-y}$$

$$U_n = 2 - \frac{1}{n^2}$$

$$U_{n+1} = 2 - \frac{1}{(n+1)^2}$$

$$U_{n+1} - U_n = \left[2 - \frac{1}{(n+1)^2}\right] - \left[2 - \frac{1}{n^2}\right]$$

$$= \frac{1}{n^2} - \frac{1}{(n+1)^2} > 0$$

b) $f(x) = \dfrac{x^2 + |x|}{|x|}$ $\quad x_0 = 0$ $\qquad ED(f) = \mathbb{R} - \{0\}$

$$|x| = \begin{cases} -x, & si \quad x < 0 \\ x, & si \quad x \geq 0 \end{cases}$$

$$\lim_{x \to 0^{>}} \frac{x^2 + |x|}{|x|} = \lim_{x \to 0^{>}} \frac{x^2 + x}{x} = \lim_{x \to 0^{>}} \frac{\cancel{x}(x+1)}{\cancel{x}} = 1$$

$$\lim_{x \to 0^{<}} \frac{x^2 + |x|}{|x|} = \lim_{x \to 0^{<}} \frac{x^2 - x}{-x} = \lim_{x \to 0^{<}} \frac{\cancel{x}(x-1)}{\cancel{-x}_{-1}} = 1$$

$$\boxed{\lim_{x \to 0} f(x) = 1}$$

d) $f(x) = \frac{|x-2|}{x^2-3x+2}$ $x_0 = 2$

$ED(f) = \mathbb{R} - \{1 ; 2\}$

$$= \frac{|x-2|}{(x-2)(x-1)}$$

$$\boxed{|x| = \begin{cases} -x, & si\ x < 0 \\ x, & si\ x \geq 0 \end{cases}}$$

$$\underbrace{|-7|}_{x} = -(-7) = 7$$

$$\lim_{x \to 2^{>}} \frac{|x-2|}{(x-2)(x-1)} = \lim_{x \to 2^{>}} \frac{\cancel{x-2}^{\,1}}{(\cancel{x-2})(x-1)} = 1$$

$$\boxed{|x-2| = \begin{cases} -(x-2), & si\ x < 2 \\ x-2, & si\ x \geq 2 \end{cases}}$$

$$|-5-2| = |-7| = -(-5-2) = 7$$

$$\lim_{x \to 2^{<}} \frac{|x-2|}{(x-2)(x-1)} = \lim_{x \to 2^{<}} \frac{\cancel{-(x-2)}^{\,-1}}{(\cancel{x-2})(x-1)} = -1$$

2.6.8 Calculer les limites suivantes :

a) $\lim_{x\to 0} \frac{\sin(2x)}{x}$

b) $\lim_{x\to 0} \frac{\sin(3x)}{\sin(2x)} = \frac{3}{2}$

$$\boxed{\lim_{x\to 0} \frac{\sin(x)}{x} = 1}$$

a) $\lim_{x\to 0} \frac{\sin(2x)}{x} \cdot \frac{2}{2}$

$= \lim_{x\to 0} \frac{\sin(2x)}{2x} \cdot 2 = 2$

b) $\lim_{x\to 0} \frac{\frac{\sin(3x)}{3x}\cdot 3}{\frac{\sin(2x)}{2x}\cdot 2} = \frac{3}{2}$

$n \neq 0 \quad \lim_{x\to 0} \frac{\sin(nx)}{nx} = 1$

g) $\lim\limits_{x\to 0} \dfrac{1-\cos(x)}{\sin^2(x)}$

h) $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\cos(x)}{x-\frac{\pi}{2}}$

g) $$\lim_{x\to 0} \frac{1-\cos(x)}{1-\cos^2(x)} = \lim_{x\to 0} \frac{\cancel{1-\cos(x)}}{\cancel{(1-\cos(x))}(1+\cos(x))} = \frac{1}{2}$$

$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos(\alpha)$

h) $$\lim_{x\to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{x-\frac{\pi}{2}} = \lim_{x\to \frac{\pi}{2}} -\frac{\sin\left(x-\frac{\pi}{2}\right)}{x-\frac{\pi}{2}} = -1$$

$\sin(-x) = -\sin(x)$