25.09.25

4.2.2 Calculer à la main :

a) $\log_3(1) = 0$ b) $\log_2(8) = 3$ c) $\log_2(64) = 6$ d) $\log_2(1'024) = 10$

e) $\log_5(5) = 1$ f) $\log_3(\sqrt{3}) = \frac{1}{2}$ g) $\log_{243}(1/243) = -1$ h) $\log_3(27) = 3$

i) $\log(1'000) = 3$ j) $\log_4(\sqrt{2}) = \frac{1}{4}$ k) $\log_{1/8}(64) = -2$ l) $\log_5(0,04) = -2$

m) $\log_3(\sqrt[4]{27}) = \frac{3}{4}$ n) $\ln(e^2) = 2$ o) $\log_a(a) = 1$ p) $\log_a(a^3) = 3$

q) $\log(10000) = 4$ r) $\ln(e) = 1$ s) $\log_2(1/8) = -3$ t) $\log_3(\sqrt[4]{3}) = \frac{1}{4}$

u) $\log(200) - \log(2)$ v) $\log_6(4) + \log_6(9)$ w) $\log_5(1) = 0$ x) $\cancel{\log(-1)}$

y) $\log(0.0001) = -4$ z) $\cancel{\ln(0)}$

j) $\log_4(\sqrt{2}) = x \iff 4^x = \sqrt{2}$

$2^{2x} = 2^{\frac{1}{2}} \Rightarrow 2x = \frac{1}{2} \Rightarrow x = \frac{1}{4}$

k) $\log_{\frac{1}{8}}(64) = x \iff \left(\frac{1}{8}\right)^x = 64$

$8^{-x} = 8^2$

Propriétés des log

Soit $a \in \mathbb{R}_+^* - \{1\}$. Soit $u, v \in \mathbb{R}$ et posons

$x = a^u$ et $y = a^v$.

① $$\log_a(xy) = \log_a(a^u \cdot a^v) = \log_a(a^{u+v})$$

$$= u + v = \log_a(x) + \log_a(y)$$

② $$0 = \log_a(1) = \log_a\left(x \cdot \frac{1}{x}\right) \overset{①}{=} \log_a(x) + \log_a\left(\frac{1}{x}\right)$$

$$\Rightarrow \log_a\left(\frac{1}{x}\right) = -\log_a(x)$$

Ex ①: $$\log(1'000'000) = \log(100 \cdot 10'000) \overset{①}{=} \log(100) + \log(10'000)$$

$$\log(1'000'000) = 6 \qquad = 2 + 4$$

Ex ②: $$\log(0{,}00001) = \log\left(\frac{1}{100'000}\right) = -\log(100'000) = -5$$

② $\Rightarrow$ ③ $$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

④ $\log_a(x^n) = \log_a\left((a^u)^n\right) = \log_a\left(a^{u\cdot n}\right) = \underbrace{u}_{\log_a(x)}\cdot n = n\cdot\log_a(x)$

Ex ④ : $\log_2(100) = \log_2(10^2) = 2\cdot\log_2(10)$

$= 2\left(\log_2(2\cdot 5)\right) = 2\left(\underbrace{\log_2(2)}_{1} + \log_2(5)\right)$

$= 2 + 2\cdot\log_2(5)$ pire beau, mais inutile ☺

⑤ Soit $b \in \mathbb{R}_+^* - \{1\}$

$$\log_a(x) = \log_a\left(b^{\log_b(x)}\right) \overset{④}{=} \log_b(x)\cdot\log_a(b)$$

$$\Rightarrow \boxed{\log_b(x) = \frac{\log_a(x)}{\log_a(b)}}$$ Changement de base

$\ln$ ou $\log = \log_{10}$

Ex ⑤ : • $\log_2(100) = \dfrac{\log_{10}(100)}{\log_{10}(2)} \cong \dfrac{2}{0,301030} \cong 6,64385$

$2^{6,64385} \cong 99,99$

• $\log_2(100) = \dfrac{\ln(100)}{\ln(2)} \cong 6,64385$

4.2.5 Résoudre les équations ci-dessous :

a) $x = \log_2(32)$ b) $2^x = 100$ c) $\log_x(256) = 4$ d) $\log_2(x) = 4$

e) $10^x = 5$ f) $e^{2x-1} = 27$ g) $\log_x(1'000) = 3$ h) $12^x = -49$

a) $x = 5 \iff 2^5 = 32$

b) $2^x = 100$ $\Rightarrow x = \underbrace{\log_2(100)}_{☹} = \underbrace{\frac{\log(100)}{\log(2)}}_{☺}$

avec la $\boxed{\text{TI}}$

$$\log(2^x) = \log(100)$$

$$x \cdot \log(2) = \log(100)$$

$$x = \frac{\log(100)}{\log(2)} = \frac{2}{\log(2)}$$

c) $\log_x(256) = 4 \iff x^4 = 256$

$$x = \sqrt[4]{256}$$

d) $\log_2(x) = 4 \iff 2^4 = x \Rightarrow x = 16$

e) $10^x = 5$ f) $e^{2x-1} = 27$ g) $\log_x(1'000) = 3$ h) $12^x = -49$ négatif

e) $x = \log(5)$

f) $2x - 1 = \ln(27)$

$x = \dfrac{\ln(27) + 1}{2}$

g) $x = 10$

h) impossible

4.2.4 Simplifier les expressions ci-dessous sans utiliser la machine :

a) $\underbrace{\log(16) + 2\log(3)} - 2\log(2) - \underbrace{\frac{1}{2}\log(9)}_{3}$ b) $\log(15) + 3\log(10) - \log(30) - \log(5)$

a) $\log\left(\frac{\cancel{16}^{4} \cdot \cancel{9}^{3}}{\cancel{3}_{1} \cdot \cancel{4}_{1}}\right) = \log(12)$

$\frac{1}{2}\log(9) = \log(\underbrace{9^{\frac{1}{2}}}_{\sqrt{9}}) = \log(3)$

4.2.6 Résoudre les équations ci-dessous :

a) $\log_{11}(x+1) = \log_{11}(7)$ b) $\log_6(2x-3) = \underbrace{\log_6(12) - \log_6(3)}_{\log_6\left(\frac{12}{3}\right)}$

a) $x + 1 = 7$

$x = 6$

b) $\log_6(2x-3) = \log_6(4)$

$2x - 3 = 4$

$x = \frac{7}{2}$ ⚠ vérification

c) $\log(x) - \log(x+1) = 3\log(4)$ d) $2\log_3(x) = 3\log_3(5)$

$\log(x) = \log(x+1) + \log(64)$

$\log(x) = \log\left(64(x+1)\right) \Rightarrow x = 64x + 64$

⚠ solution parasite

$63x = -64$

$x = -\frac{64}{63}$

On doit vérifier que cette solution satisfait l'équation de départ.

$x = \frac{-64}{63}$ ne convient pas, $\log\left(\frac{-64}{63}\right)$ n'existe pas. $S = \emptyset$

d) $\log_3(x^2) = \log_3(5^3) \Rightarrow x^2 = 125$

$\Rightarrow x = \pm\sqrt{125}$, on exclut la valeur négative

$\Rightarrow S = \{\sqrt{125}\}$

e) $\underbrace{\ln(x) + \ln(x-2)} = 0,5\ln(9)$ f) $\log_8(x+4) = 1 - \log_8(x-3)$

⚠ vérifier les solutions

e) $\ln\left(x(x-2)\right) = \ln(3)$

$x(x-2) = 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

- $\underline{x = 3}$: $\ln(3) + \underbrace{\ln(1)}_{0} \stackrel{?}{=} \ln(3)$ ✓
- $\underline{x = -1}$: $\ln(-1)$ ✗

$S = \{3\}$

f) $\log_8(x+4) = 1 - \log_8(x-3)$

$\log_8(x+4) = \log_8(8) - \log_8(x-3)$

$\log_8(x+4) + \log_8(x-3) = \log_8(8)$

$\log_8\left((x+4)(x-3)\right) = \log_8(8)$

$\Rightarrow (x+4)(x-3) = 8$

...