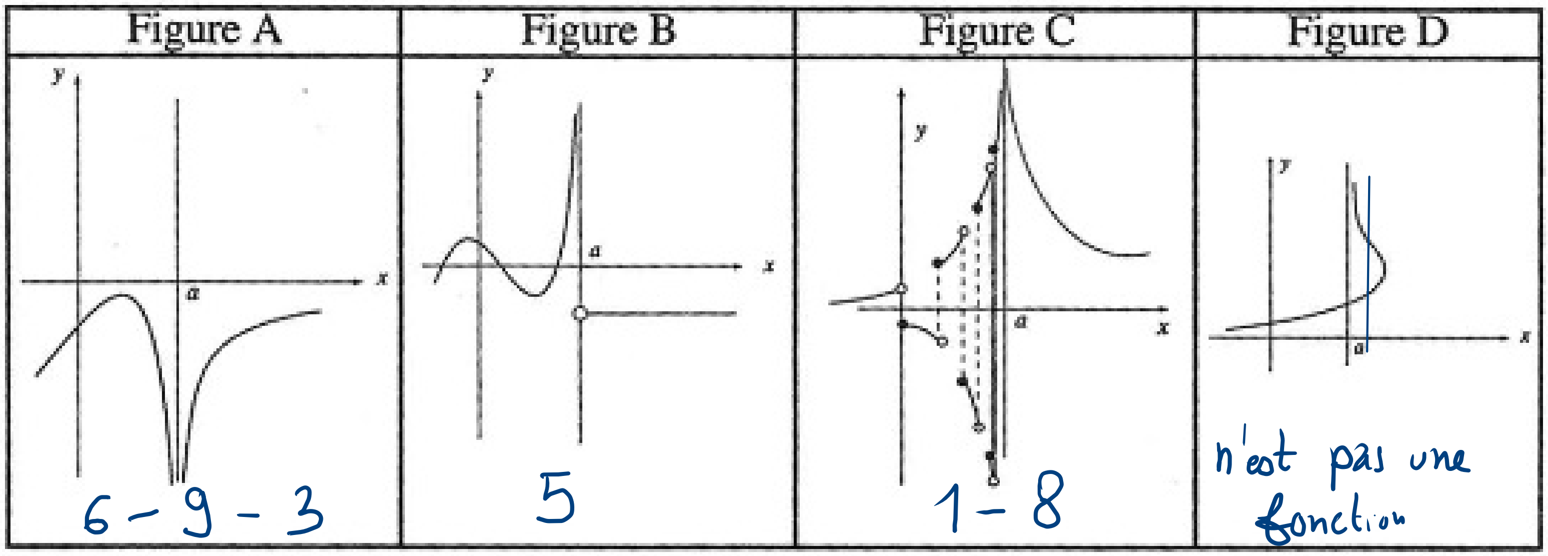
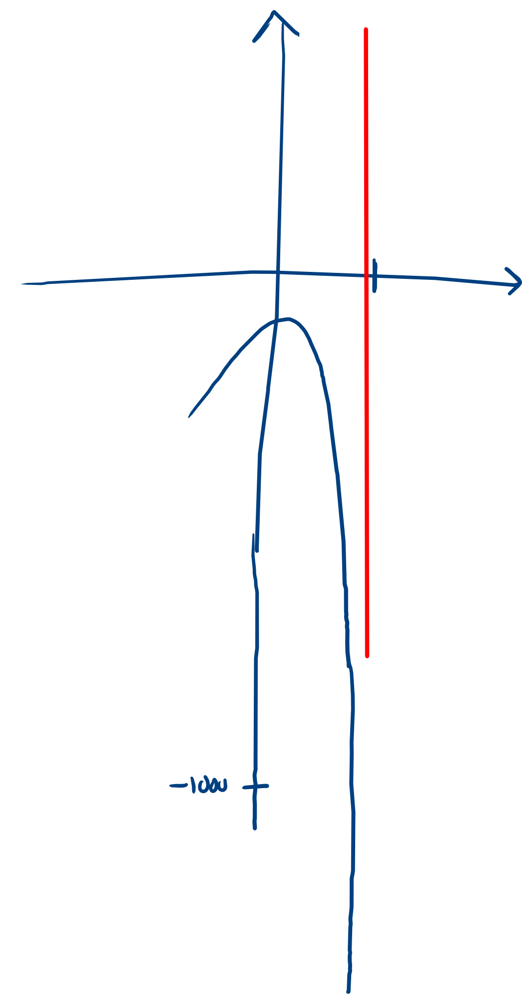
27.11.25

2.6.13 Dire pour chacune des quatre figures ci-dessous quelles sont les notations autorisées parmi 1), 2), . . . , 9) :



$$\lim_{x\to a} f(x) = \begin{cases} 1) & \infty \\ 2) & +\infty \\ 3) & -\infty \end{cases} \qquad \lim_{\substack{x\to a \\ <}} f(x) = \begin{cases} 4) & \infty \\ 5) & +\infty \\ 6) & -\infty \end{cases} \qquad \lim_{\substack{x\to a \\ >}} f(x) = \begin{cases} 7) & \infty \\ 8) & +\infty \\ 9) & -\infty \end{cases}$$



2.6.14 Calculer les limites suivantes :

a) $\displaystyle\lim_{x\to-2}\frac{x^2+3x+6}{(x+2)^2}$

b) $\displaystyle\lim_{x\to-3}\frac{x^2+2x-15}{x^2+8x+15}$

c) $\displaystyle\lim_{x\to0}\frac{x^2-3x}{x^3}$

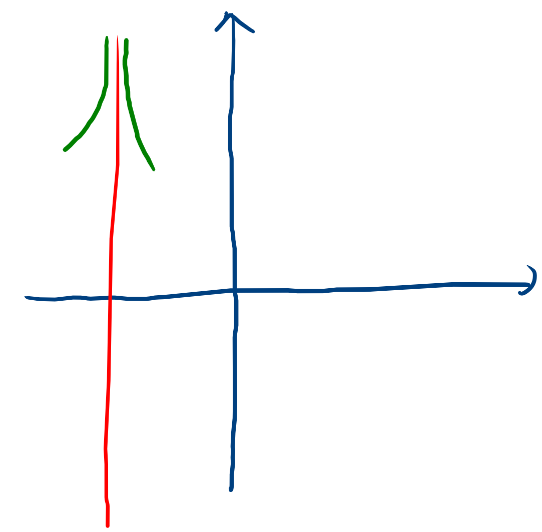
d) $\displaystyle\lim_{\substack{x\to5\\>}}\frac{x-3}{5-x}$

e) $\displaystyle\lim_{x\to1}(2x^2-5x+3)\frac{1}{x-1}$

f) $\displaystyle\lim_{x\to1}\left(\frac{x^2}{x-1}-\frac{1}{x-1}\right)$

g) $\displaystyle\lim_{\substack{x\to-2\\<}}\frac{x-1}{x+2}$

h) $\displaystyle\lim_{x\to2}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)$



a) $$\lim_{x\to\underbrace{-2}_{\notin ED(f)}}\underbrace{\frac{x^2+3x+6}{(x+2)^2}}_{f(x)} = +\infty \qquad \text{“}\frac{4}{0}\text{”}$$

$$\lim_{\substack{x\to-2\\<}}\frac{x^2+3x+6}{(x+2)^2} = +\infty \qquad \text{“}\frac{4}{0_+}\text{”}$$

$$\lim_{\substack{x\to-2\\>}}\frac{x^2+3x+6}{(x+2)^2} = +\infty \qquad \text{“}\frac{4}{0_+}\text{”}$$

$$\lim_{x \to 0} \frac{1}{x} \underset{\text{"}\frac{1}{0}\text{"}}{=} \infty$$

$$\lim_{x \to 0} \frac{x^2}{x} \underset{\text{"}\frac{0}{0}\text{"}}{=} 0$$

$$\lim_{x \to 0} \frac{x}{x^2} \underset{\text{"}\frac{0}{0}\text{"}}{=} \infty$$

b) $\underbrace{\lim_{x \to -3} \frac{x^2 + 2x - 15}{x^2 + 8x + 15}}_{\text{fonction}} = \infty$

" $\frac{-12}{0}$ "

<u>1ère m:</u> $\lim_{x \to -3} \frac{\cancel{(x+5)}(x-3)}{\cancel{(x+5)}(x+3)} = \lim_{x \to -3} \frac{x-3}{x+3}$ " $\frac{-6}{0}$ "

$\lim_{x \xrightarrow{<} -3} f(x) = +\infty$

$\lim_{x \xrightarrow{>} -3} f(x) = -\infty$

<u>2ème m:</u> Tableau des signes

x		-5		-3		3	
$f(x)$	$+$	‖	$+$	‖	$-$	0	$+$

c) $\lim_{x \to 0} \frac{x^2 - 3x}{x^3}$ $\overset{\text{Ind}}{=}$ " $\frac{0}{0}$ " $\lim_{x \to 0} \frac{\cancel{x}(x-3)}{\cancel{x^3}_{x^2}} = \lim_{x \to 0} \frac{x-3}{x^2} = -\infty$ " $\frac{-3}{0}$ "

d) $\lim\limits_{\substack{x \to 5 \\ >}} \dfrac{x-3}{5-x} = -\infty$

$$\text{“}\frac{2}{0_-}\text{”}$$

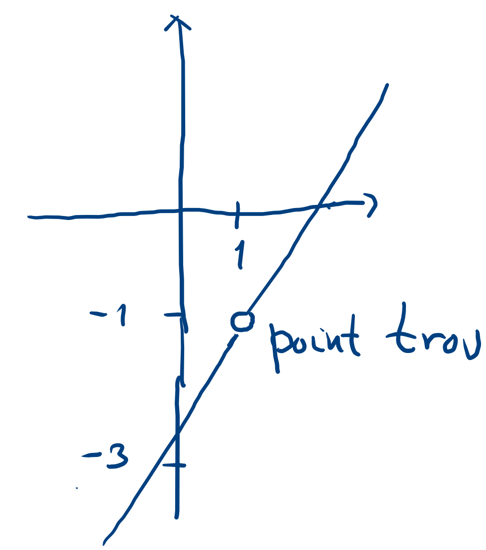
e) $\lim\limits_{x \to 1} (2x^2 - 5x + 3) \dfrac{1}{x-1} \overset{\text{Ind}}{=} \lim\limits_{x \to 1} \dfrac{2x^2 - 5x + 3}{x-1} = \lim\limits_{x \to 1} \dfrac{\cancel{(x-1)}(2x-3)}{\underbrace{\cancel{x-1}}_{1}}$

$$\text{“}0 \cdot \infty\text{”}$$

$$= -1$$

$$\boxed{\frac{0}{0} \ ; \ \frac{\infty}{\infty} \ ; \ \infty \cdot 0 \ ; \ \infty - \infty \ \text{ Formes indéterminées}}$$

e) $ED(f) = \mathbb{R} - \{1\}$ point trou $(1 ; -1)$



f) $\lim_{x\to 1}\left(\frac{x^2}{x-1}-\frac{1}{x-1}\right) \underset{\infty-\infty}{\overset{\text{Ind}}{=}} \lim_{x\to 1} \frac{x^2-1}{x-1} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\text{Ind}}{=}} \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = 2$

point trou $(1;2)$

g) $\lim_{\substack{x\to -2 \\ <}} \frac{x-1}{x+2} \underset{\text{"}\frac{-3}{0_-}\text{"}}{=} +\infty$

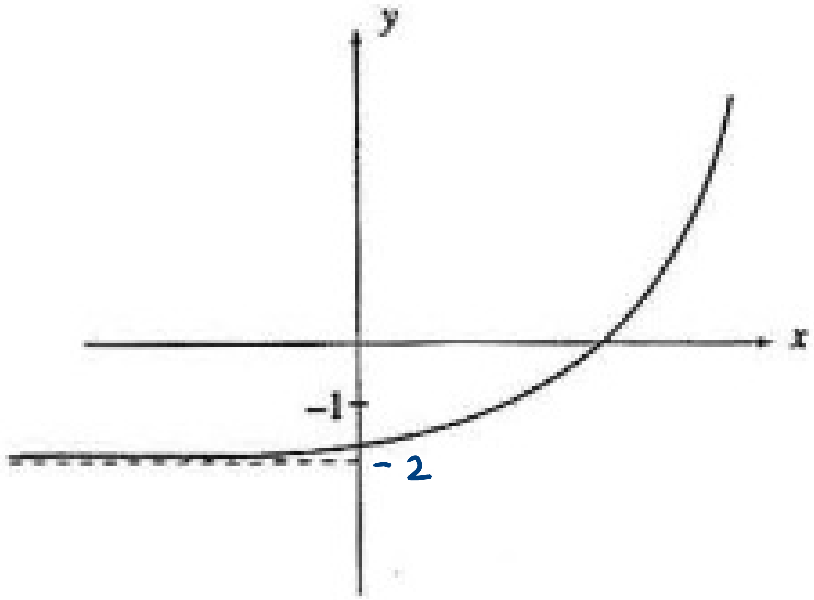
h) $\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right) \underset{\text{"}\infty-\infty\text{"}}{\overset{\text{Ind}}{=}} \lim_{x\to 2}\left(\frac{1}{x-2}-\frac{4}{(x-2)(x+2)}\right)$

$= \lim_{x\to 2} \frac{x+2-4}{(x-2)(x+2)} = \lim_{x\to 2} \frac{\overset{1}{\cancel{x-2}}}{\cancel{(x-2)}(x+2)} = \frac{1}{4}$

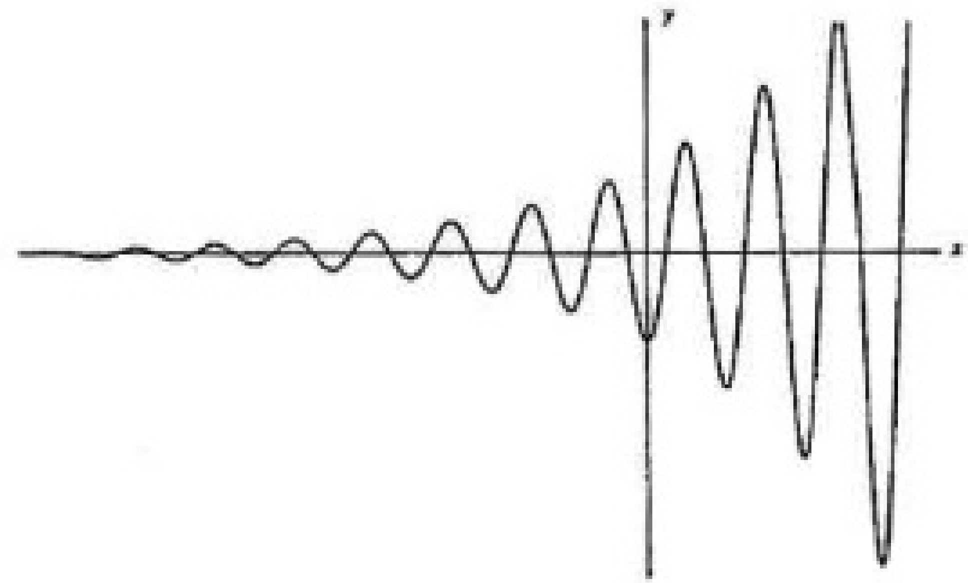
point trou $(2;\frac{1}{4})$

2.6.15 Lire les limites : $\lim_{x \to -\infty} f(x)$, $\lim_{x \to +\infty} f(x)$ et $\lim_{x \to \infty} f(x)$.

a)



b)



$\lim_{x \to -\infty} f(x) = -2$

$\lim_{x \to +\infty} f(x) = \infty$

$\cancel{\lim_{x \to \infty} f(x)}$.

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to +\infty} f(x)$ –

$\cancel{\lim_{x \to \infty} f(x)}$.

2.6.16 Calculer les limites suivantes :

a) $\lim\limits_{x\to\infty} \dfrac{2x-4}{-3x+1}$ b) $\lim\limits_{x\to-\infty} \dfrac{-3x^2+1}{x+2}$

a) $$\lim_{x\to\infty} \frac{2x-4}{-3x+1} \underset{\text{“}\frac{\infty}{\infty}\text{”}}{=} \lim_{x\to\infty} \frac{2\cancel{x}}{-3\cancel{x}} = -\frac{2}{3}$$

b) $$\lim_{x\to-\infty} \frac{-3x^2}{x} = \lim_{x\to-\infty} -3x = +\infty$$

c) $\lim\limits_{x\to\infty} \dfrac{x^2+4x+29}{x^2-2x+4}$ d) $\lim\limits_{x\to\infty} \dfrac{(3x+4)(x-1)}{(2x+7)(1-5x)}$

c) $$= \lim_{x\to\infty} \frac{x^2+4x}{x^2-2x} = \lim_{x\to\infty} \frac{\cancel{x}(x+4)}{\cancel{x}(x-2)} = \lim_{x\to\infty} \frac{x+4}{x-2}$$

$$= \lim_{x\to\infty} \frac{\overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}}} = 1$$

$$\lim_{x\to\infty} \frac{\cancel{x^2}\left(1+\frac{4}{x}+\frac{29}{x^2}\right)}{\cancel{x^2}\left(1-\frac{2}{x}+\frac{4}{x^2}\right)} = 1$$

$$\lim_{x\to\infty} \frac{a_m x^m + \dots}{b_n x^n + \dots} = \begin{cases} \infty & \text{si } m>n \\ \dfrac{a_m}{b_n} & \text{si } m=n \\ 0 & \text{si } m<n \end{cases}$$

Limites à l'infini de fonctions rationnelles

$$\lim_{x\to\pm\infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0} = \lim_{x\to\pm\infty} \frac{a_n x^n}{b_m x^m} = \begin{cases} 0 & \text{si } n<m \\ \dfrac{a_n}{b_m} & \text{si } n=m \\ \infty & \text{si } n>m \end{cases}$$

Formulaire Burier

e) $$\lim_{x\to\infty} \frac{(x+1)^7(2x+3)^4}{(2x+1)^3(x-98)^8} = \lim_{x\to\infty} \frac{16x^{11}+\dots}{8x^{11}+\dots} = 2$$