1.1.4 Résoudre dans $\mathbb{C}$ le système d'équations ci-dessous :

$$\begin{cases} (2+i)z + (2-i)w &= 7-4i \\ (1+i)z - iw &= 2+i \end{cases}$$

$$\left\{\begin{array}{l} (2+i)\,z + (2-i)\,w = 7-4i \\ (1+i)\,z - i\,w = 2+i \end{array}\right. \quad \begin{array}{|l} w \\ \cdot\, i \\ \cdot\, (2-i) \end{array}$$

① Déterminons z :

$$\left\{\begin{array}{l} (2i-1)\,z + (2i+1)\,w = 7i+4 \\ (3+i)\,z - (2i+1)\,w = 5 \end{array}\right. \qquad \left\{ (1+i)(2-i) = 2+1+i \right.$$

$$(2+3i)\,z = 9+7i \qquad \Big|\ \div (2+3i)$$

$$z = \frac{9+7i}{2+3i} \cdot \frac{2-3i}{2-3i}$$

$$z = \frac{18+21-13i}{13} = \frac{39-13i}{13} = \frac{39}{13} - \frac{13}{13}i = \underline{3-i}$$

② Déterminons w par substitution :

$$(1+i)(3-i) - iw = 2+i$$

$$4+2i - iw = 2+i$$

$$iw = 2+i \qquad \Big|\ \div i$$

$$w = \frac{2+i}{i} \cdot \frac{i}{i} = \frac{-1+2i}{-1} = \underline{1-2i}$$

$$\begin{cases} z = 3-i \\ w = 1-2i \end{cases}$$

1.2.2 Écrire les nombres complexes ci-dessous sous forme trigonométrique :

a) $1 = z_1 = [1; 0]$

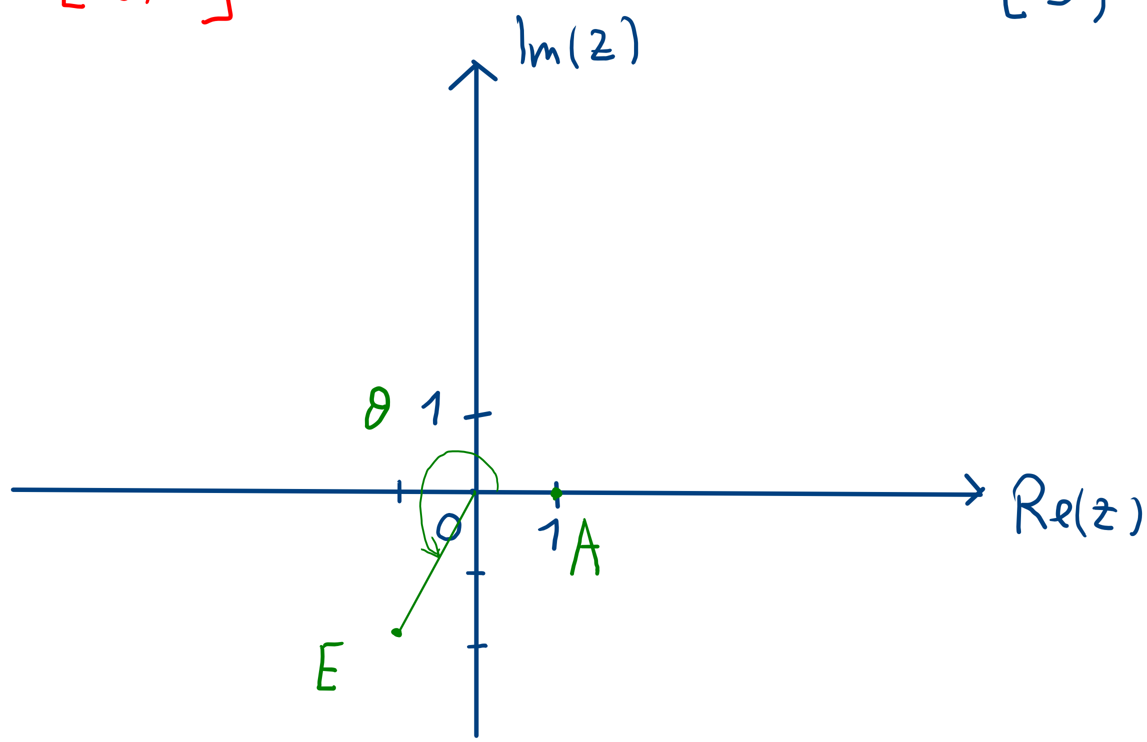
b) $i = [1; \frac{\pi}{2}]$

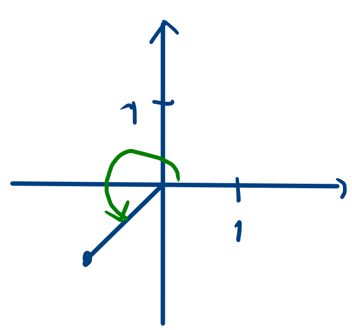
c) $-2 = [2; \pi]$

d) $-1-i = [\sqrt{2}; \frac{5\pi}{4}]$

e) $-1-\sqrt{3}i$ E $[2; \frac{4\pi}{3}]$

f) $3+4i = [5; 53°]$

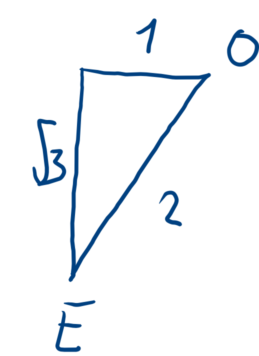




e) $z_5 = -1-\sqrt{3}\,i$

$|z_5| = \sqrt{1+3} = 2$

$$\begin{cases} \cos(\theta) = \frac{-1}{2} \\ \sin(\theta) = \frac{-\sqrt{3}}{2} \end{cases}$$



$\theta = 240° \cdot \frac{\pi}{180°} = \frac{4\pi}{3}$

1.2.3 Écrire les nombres complexes ci-dessous sous forme algébrique :

a) $\left[4; -\frac{\pi}{3}\right]$ $\overline{\underline{\text{IV}}}$

b) $\left[\frac{3}{4}; \frac{3\pi}{4}\right]$ II

c) $[\pi; -\pi]$ axe des Re

$\quad = -\pi$

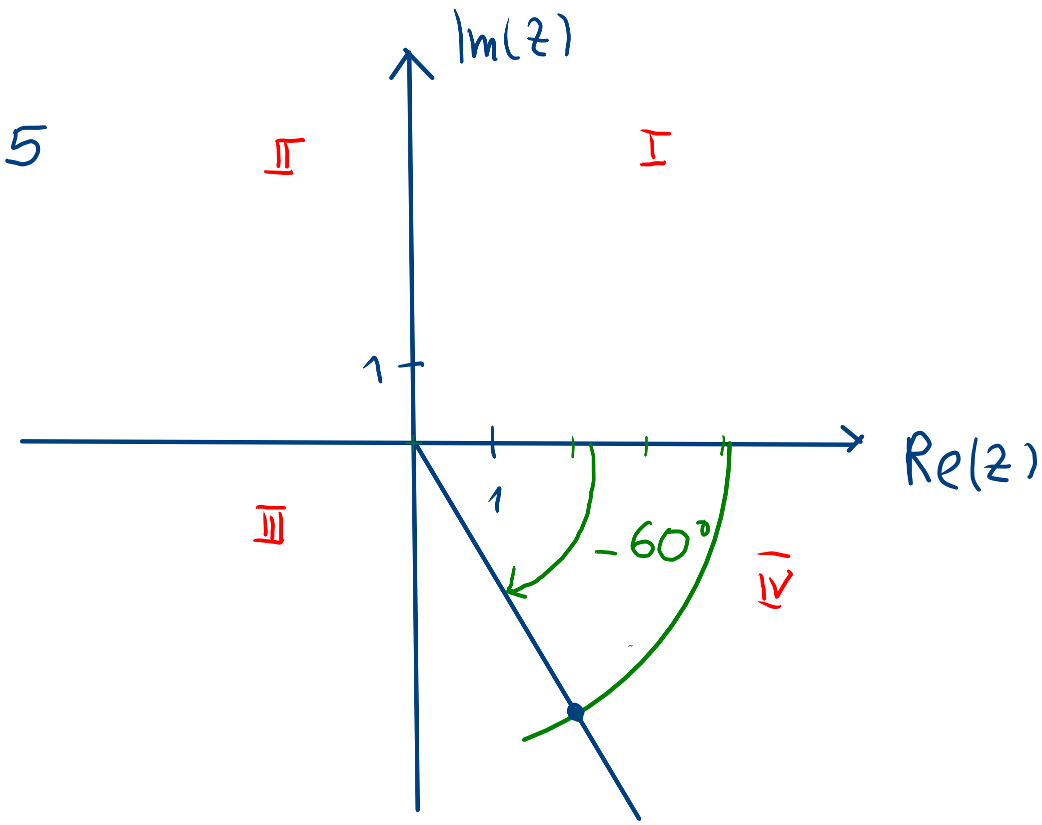
d) $\left[4; \frac{\pi}{3}\right]$ I

e) $\left[1; -\frac{\pi}{2}\right]$ axe des Im $= -i$

[f) $e^{i\pi}$ plus tard]

$$z = \underset{\substack{\uparrow \\ |z|}}{r}\left(\cos(\theta) + \sin(\theta)\, i\right)$$

$[5; -\pi] = -5$



a) $z = 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = 2 - 2\sqrt{3}\, i$

$$\begin{cases} \cos(-60°) = \frac{1}{2} \\ \sin(-60°) = -\sin(60°) = -\frac{\sqrt{3}}{2} \end{cases}$$

Formulaire

b) $z = \frac{3}{4}\left(\cos\left(\frac{3\pi}{4}\right) + \sin\left(\frac{3\pi}{4}\right) i\right) = \frac{3}{4}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right) = \frac{-3\sqrt{2}}{8} + \frac{3\sqrt{2}}{8} i$

d) $\left[4; \frac{\pi}{3}\right] = 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = 2 + 2\sqrt{3}\, i$

1.2.4 Calculer :

$\frac{\pi}{4}+\frac{\pi}{6}=\frac{5\pi}{12}$

a) $\underbrace{\left[2;\frac{\pi}{4}\right]}_{z_1}\cdot\underbrace{\left[3;\frac{\pi}{6}\right]}_{z_2} = \left[6;\frac{5\pi}{12}\right]$ b) $\left[6;\frac{2\pi}{3}\right]:\left[3;-\frac{\pi}{3}\right]$ c) $\left[2;\frac{\pi}{3}\right]^3 = \left[8,\pi\right] = -8$

a) $z_1 = 2\left(\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)i\right) = 2\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$

$z_2 = 3\left(\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{6}\right)i\right) = 3\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$

$z_1\cdot z_2 = 6\left[\underbrace{\left(\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}\right)}_{\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}+\left(\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}\right)i\right]$

Formule : $[r_1,\theta_1]\cdot[r_2,\theta_2] = [r_1\cdot r_2\,;\,\theta_1+\theta_2]$

b) $[2\,;\,\pi] = -2$

Formule de Moivre

$$\boxed{\big(\cos(\varphi)+i\sin(\varphi)\big)^n = \cos(n\varphi)+i\sin(n\varphi)}$$

$[r,\theta]^n = [r^n\,;\,n\theta]$

1.2.5 Calculer $z_1\, z_2$ et z_1/z_2 en utilisant la forme trigonométrique :

a) $z_1 = -1 + i, \quad z_2 = 1 + i$

b) $z_1 = -2 - 2\sqrt{3}\, i, \quad z_2 = 5\, i$

c) $z_1 = 2\, i, \quad z_2 = -3\, i$

d) $z_1 = -10, \quad z_2 = -4$

$$z_1 = [r_1, \theta_1]$$

$$z_2 \quad [r_2, \theta_2]$$

$$\frac{z_1}{z_2} = \left[\frac{r_1}{r_2}, \theta_1 - \theta_2\right]$$