$\tan(\alpha) = \frac{2\sqrt{3}}{2} \Rightarrow \alpha = \frac{\pi}{3}$

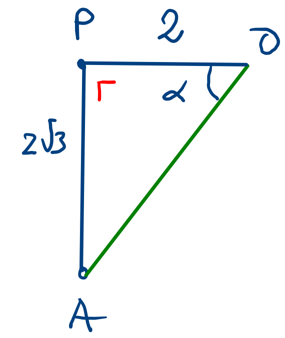
1.2.5 Calculer $z_1\, z_2$ et z_1/z_2 en utilisant la forme trigonométrique :

a) $z_1 = -1 + i, \quad z_2 = 1 + i$

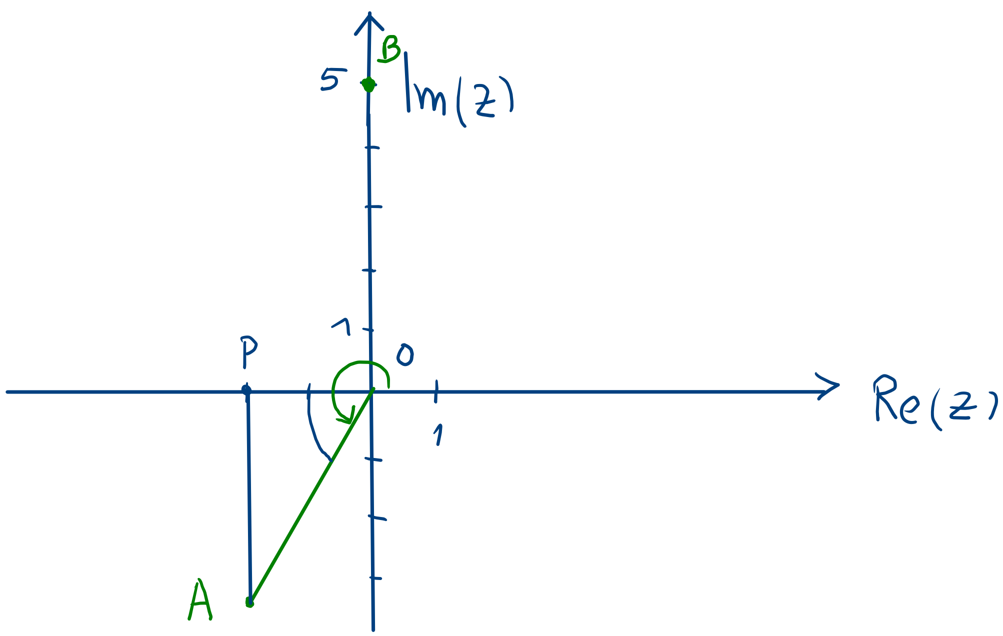
b) $z_1 = \overset{A}{-2 - 2\sqrt{3}\, i}, \quad z_2 = \overset{B}{5\, i}$

c) $z_1 = 2\, i, \quad z_2 = -3\, i$

d) $z_1 = -10, \quad z_2 = -4$



b)



- $z_1 = -2 - 2\sqrt{3}\, i$

$|z_1| = \sqrt{(-2)^2 + (-2\sqrt{3})^2}$

$= \sqrt{4 + 12} = 4$

$z_1 = [4; \frac{4\pi}{3}]$

- $z_2 = [5; \frac{\pi}{2}]$

$\frac{4\pi}{3} + \frac{\pi}{2} = \frac{8\pi}{6} + \frac{3\pi}{6}$

$$z_1 \cdot z_2 = [4; \tfrac{4\pi}{3}] \cdot [5; \tfrac{\pi}{2}] = [20; \tfrac{11\pi}{6}]$$

$$= 20\left(\cos\left(\tfrac{11\pi}{6}\right) + \sin\left(\tfrac{11\pi}{6}\right) i\right)$$

$$= 20\left(\tfrac{\sqrt{3}}{2} - \tfrac{1}{2} i\right) = 10\sqrt{3} - 10\, i$$

$$\frac{z_1}{z_2} = [4; \tfrac{4\pi}{3}] \div [5; \tfrac{\pi}{2}] = [\tfrac{4}{5}; \tfrac{5\pi}{6}] = \tfrac{4}{5}\left(\cos\left(\tfrac{5\pi}{6}\right) + \sin\left(\tfrac{5\pi}{6}\right) i\right)$$

$$= \tfrac{4}{5}\left(-\tfrac{\sqrt{3}}{2} + \tfrac{1}{2} i\right) = -\tfrac{2\sqrt{3}}{5} + \tfrac{2}{5}\, i$$

1.2.9 Déterminer sous forme trigonométrique et représenter dans le plan complexe :

a) les racines septièmes de l'unité,

b) les solutions de l'équation $z^5 = -32$.

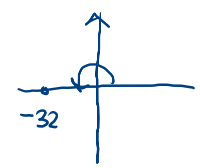
b) Equation réelle : $x^5 = -32$

$x = -2$

Equation complexe : $z^5 = -32$

Théorème fondamental de l'algèbre

Toute équation complexe de degré n admet exactement n solutions



On écrit -32 sous sa forme trigo : $-32 = [32 ; \pi]$

On cherche $z = [r, \theta]$ tel que $z^5 = -32$

Donc $[r^5 ; 5\theta] = [32 ; \pi]$

$$\begin{cases} r^5 = 32 & \Rightarrow \quad r = 2 \\ 5\theta = \pi + k \cdot 2\pi & \Rightarrow \quad \theta = \dfrac{\pi}{5} + k \cdot \dfrac{2\pi}{5} \end{cases}$$

1) $k = 0$ $\quad \theta = \frac{\pi}{5} \quad \Rightarrow \quad z_1 = [2 ; \frac{\pi}{5}] \quad \Rightarrow \quad z_1 = [2 ; 36°]$

2) $k = 1$ $\quad \theta = \frac{\pi}{5} + \frac{2\pi}{5} = \frac{3\pi}{5} \quad \Rightarrow \quad z_2 = [2, \frac{3\pi}{5}] \quad \Rightarrow \quad z_2 = [2 ; 108°]$

3) $k = 2$ $\quad \theta = \frac{\pi}{5} + \frac{4\pi}{5} = \pi \quad \Rightarrow \quad z_3 = [2 ; \pi] = -2 \quad \quad z_3 = [2 ; 180°]$

4) $k = 3$ $\quad \theta = \frac{\pi}{5} + \frac{6\pi}{5} = \frac{7\pi}{5} \quad \Rightarrow \quad z_4 = [2 ; \frac{7\pi}{5}] \quad \quad z_4 = [2 ; 252°]$

5) $k = 4$ $\quad \theta = \frac{\pi}{5} + \frac{8\pi}{5} = \frac{9\pi}{5} \quad \Rightarrow \quad z_5 = [2 ; \frac{9\pi}{5}] \quad \quad z_5 = [2 ; 324°]$

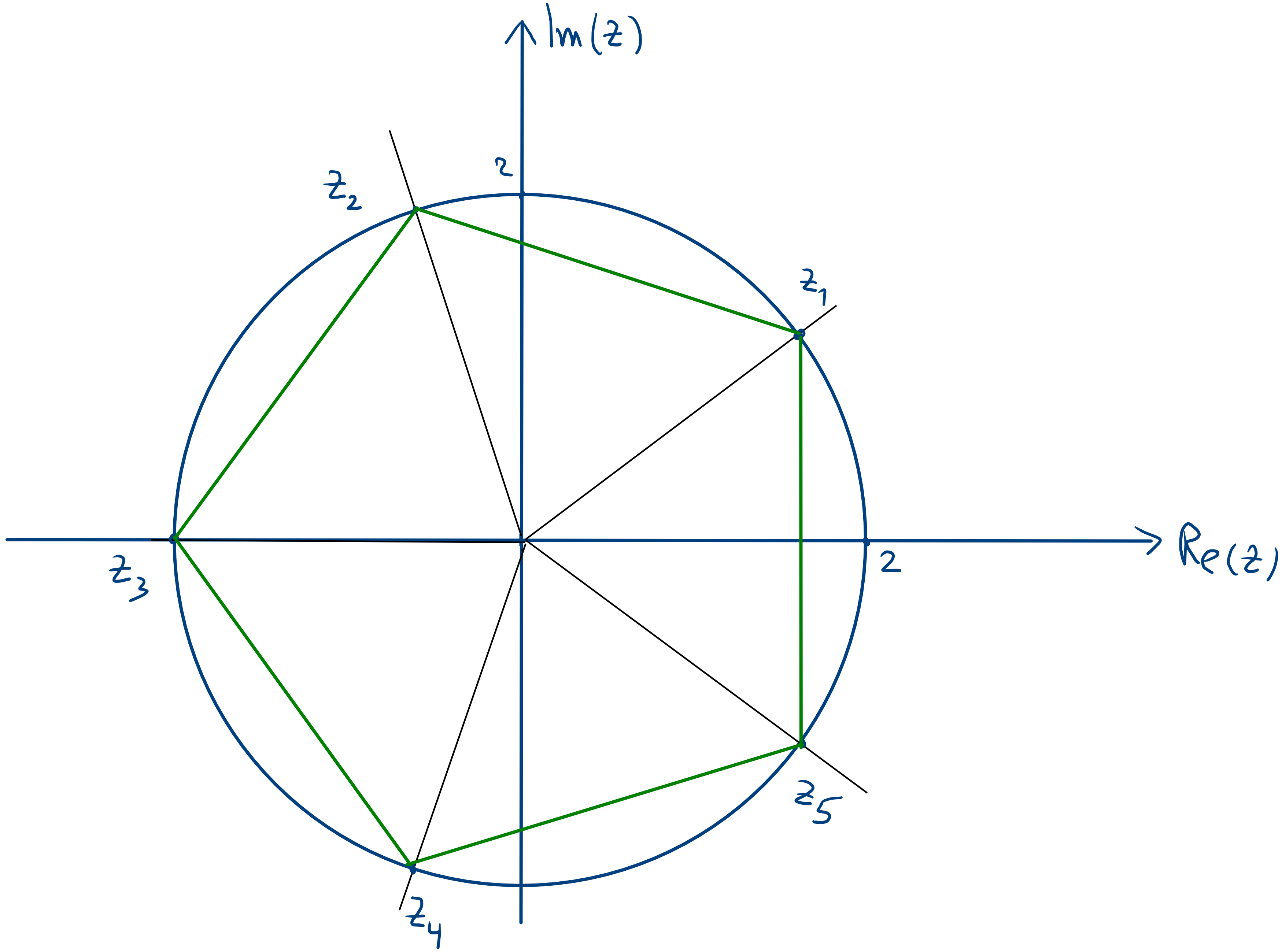
$z_1 = [2;\ 36°]$

$z_2 = [2;\ 108°]$

$z_3 = [2;\ 180°]$

$z_4 = [2;\ 252°]$

$z_5 = [2;\ 324°]$



Les solutions complexes de l'équation forment un pentagone régulier.