

Série 1 – Nombres complexes**Exercice 1**Calculer. Donner les résultats sous la forme $a + bi$.

a) $(5 - 8i) - (12 + 3i) + (7 - 7i) =$

d) $\frac{3 + 6i}{3 - 4i} =$

b) $(5 - 2i)(5 + 2i) =$

e) $\frac{2 - 3i}{4 - 3i} + \frac{2 + 3i}{4 + 3i} =$

c) $(1 - 2i)^3 =$

a) $(5 - 8i) - (12 + 3i) + (7 - 7i) =$

$$\underline{5 - 8i} - \underline{12 + 3i} + \underline{7 - 7i} = -18i$$

b) $(5 - 2i)(5 + 2i) =$

$$25 - 4i^2 = 25 + 4 = 29$$

c) $(1 - 2i)^3 =$

$$\begin{aligned} (1 - 2i)^2 &= 1 - 4i + 4i^2 = 1 - 4i - 4 = -3 - 4i \\ (1 - 2i)(-3 - 4i) &= -3 + 8i^2 - 4i + 6i \\ &= -11 + 2i \end{aligned}$$

d) $\frac{3 + 6i}{3 - 4i} =$

$$\begin{aligned} \frac{3 + 6i}{3 - 4i} \cdot \frac{3 + 4i}{3 + 4i} &= \frac{9 + 24i^2 + 12i + 18i}{9 + 16} = \frac{-15 + 30i}{25} \\ &= -\frac{3}{5} + \frac{6}{5}i \end{aligned}$$

e) $\frac{2 - 3i}{4 - 3i} + \frac{2 + 3i}{4 + 3i} =$

$$\begin{aligned} &\frac{(2 - 3i)(4 + 3i) + (2 + 3i)(4 - 3i)}{16 + 9} \\ &= \frac{8 - 9i^2 - 12i + 6i + 8 - 9i^2 + 12i - 6i}{25} = \frac{34}{25} \end{aligned}$$

Exercice 2

Soit les nombres complexes : $z_1 = \frac{3-i}{5+7i}$ et $z_2 = \frac{3+i}{5-7i}$.

Calculer $z_1 - \overline{z_2}$, $z_1 + z_2$ et $z_1 - z_2$.

$$z_1 = \frac{(3-i)(5-7i)}{25+49} = \frac{8-26i}{74} = \frac{4}{37} - \frac{13}{37}i$$

$$z_2 = \frac{(3+i)(5+7i)}{25+49} = \frac{8+26i}{74} = \frac{4}{37} + \frac{13}{37}i$$

On voit que $\overline{z_1} = z_2$.

$$\bullet \quad z_1 - \overline{z_2} = 0$$

$$\bullet \quad z_1 + z_2 = \frac{8}{37}$$

$$\bullet \quad z_1 - z_2 = -\frac{26}{37}i$$

Exercice 3

Soit $z_1 = -1 + 2i$ et $z_2 = 1 - i$. Ecrire sous forme algébrique les nombres complexes :

$$\bullet z_1^2 - 2z_2 \quad \bullet z_1 z_2^2 \quad \bullet \frac{z_1}{z_2} \quad \bullet \frac{1}{z_1} + \frac{1}{z_2} \quad \bullet \frac{1}{z_1^2} + \frac{1}{z_2^2}$$

$$\bullet z_1^2 - 2z_2 = \underline{-5 - 2i}$$

$$\bullet z_1 z_2^2 = (-1 + 2i)(-2i) = \underline{4 + 2i}$$

$$\bullet \frac{z_1}{z_2} = \underline{-\frac{3}{2} + \frac{1}{2}i}$$

$$\bullet \frac{1}{z_1} = -\frac{1}{5} - \frac{2}{5}i$$

$$\frac{1}{z_2} = \frac{1}{2} + \frac{1}{2}i$$

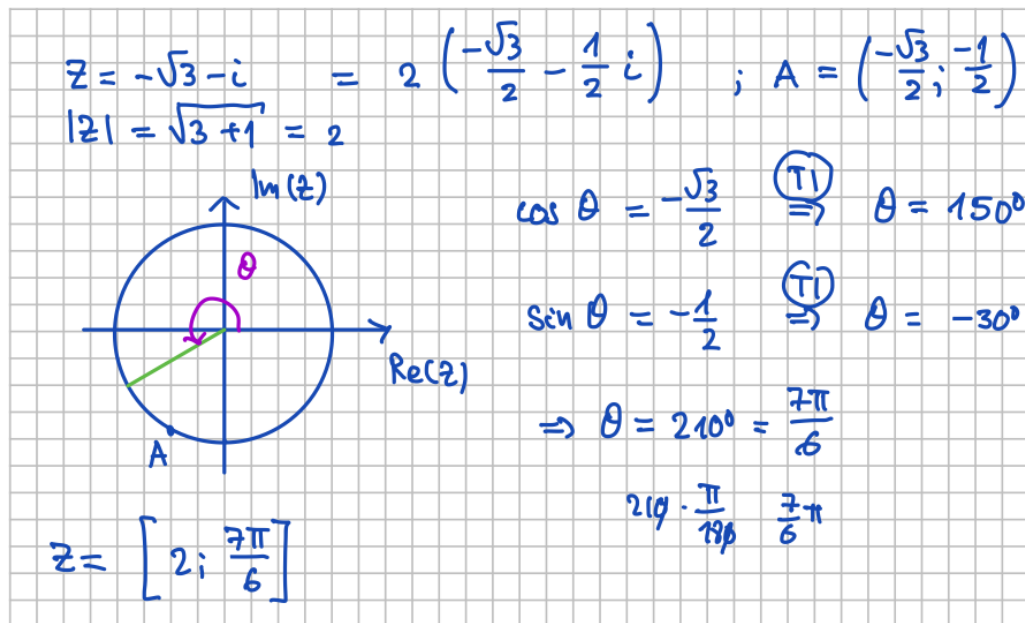
$$\frac{1}{z_1} + \frac{1}{z_2} = \underline{\frac{3}{10} + \frac{1}{10}i}$$

$$\bullet \frac{1}{z_1^2} + \frac{1}{z_2^2} = \left(-\frac{3}{25} + \frac{4}{25}i\right) + \left(\frac{1}{2}i\right) = \underline{-\frac{3}{25} + \frac{33}{50}i}$$

Exercice 4

Ecrire sous forme trigonométrique le nombre complexe suivant :

$$z = -\sqrt{3} - i$$

**Exercice 5**

Ecrire le nombre complexe $\omega = \left[2 ; \frac{5\pi}{6} \right]$ sous forme algébrique.

$$\begin{aligned}
 \omega &= 2 \left(\cos\left(\frac{5\pi}{6}\right) + \sin\left(\frac{5\pi}{6}\right)i \right) \\
 &= 2 \left(\cos(150^\circ) + \sin(150^\circ)i \right) \\
 &= 2 \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) \\
 &= -\sqrt{3} + i
 \end{aligned}$$