

Exercise 1

$$a) \frac{3+6i}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{9-24+30i}{25} = \frac{-15+30i}{25} = -\frac{3}{5} + \frac{6}{5}i$$

$$b) \frac{\cos(t) + \sin(t)i}{\cos(t) - \sin(t)i} \cdot \frac{\cos(t) + \sin(t)i}{\cos(t) + \sin(t)i} = \frac{\cos^2(t) - \sin^2(t) + 2\cos(t)\sin(t)i}{\cos^2(t) + \sin^2(t)}$$

$$c) \left[\begin{aligned} \frac{(1+i)^2}{(2-i)^2} &= \frac{2i}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{-8+6i}{25} \\ \frac{3+6i}{3-4i} \cdot \frac{3+4i}{3+4i} &= \frac{-15+30i}{25} \end{aligned} \right] \quad \frac{-8+6i-15+30i}{25} = \frac{-23+36i}{25}$$

$$d) \frac{(2+5i)(1+i) + (2-5i)(1-i)}{2} = \frac{-3+7i-3-7i}{2} = -3$$

Exercise 2

$$a) z = [2; \frac{\pi}{3}] = 2 \left(\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)i \right) = 2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = 1 + \sqrt{3}i$$

$$b) z = [3; -\frac{3\pi}{4}] = 3 \left(\cos\left(-\frac{3\pi}{4}\right) + \sin\left(-\frac{3\pi}{4}\right)i \right) = 3 \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right)$$

Exercice 3

$$a) z = 3 - 4i = (a + bi)^2 = a^2 - b^2 + 2abi$$

$$\begin{cases} a^2 - b^2 = 3 \\ 2ab = -4 \\ a^2 + b^2 = 5 \end{cases} \begin{array}{c|c|c} \cdot 1 & \cdot (-1) & \\ \hline \cdot 1 & \cdot 1 & \end{array} \begin{cases} a^2 = 4 \\ b^2 = 1 \\ 2ab = -4 \end{cases} \begin{cases} a = \pm 2 \\ b = \pm 1 \\ ab = -2 \end{cases}$$

les racines carrées de $3 - 4i$ sont : $2 - i$ et $-2 + i$

$$b) z = 24 - 10i = (a + bi)^2 = a^2 - b^2 + 2abi$$

$$\begin{cases} a^2 - b^2 = 24 \\ 2ab = -10 \\ a^2 + b^2 = 26 \end{cases} \begin{array}{c|c|c} \cdot 1 & \cdot (-1) & \\ \hline \cdot 1 & \cdot 1 & \end{array} \begin{cases} a^2 = 25 \\ b^2 = 1 \\ ab = -5 \end{cases} \begin{cases} a = \pm 5 \\ b = \pm 1 \\ ab = -5 \end{cases}$$

les racines carrées de $24 - 10i$ sont : $5 - i$ et $-5 + i$

Exercice 4

Soit l'équation $z^3 = \underbrace{\frac{1}{4}(-1+ji)}_w$. $z = [r; \theta]$
 $z^3 = [r^3, 3\theta]$

$$w = \left[\frac{\sqrt{2}}{4}; \frac{3\pi}{4} \right] = \frac{1}{4} \left[\sqrt{2}; \frac{3\pi}{4} \right]$$

$$r^3 = \frac{\sqrt{2}}{4} \Rightarrow r = \sqrt[3]{\frac{\sqrt{2}}{4}} = \sqrt[3]{\sqrt{\frac{2}{16}}} = \sqrt[3]{\sqrt{\frac{1}{8}}} = \sqrt[2]{\sqrt[3]{8}} = \frac{1}{\sqrt{2}}$$

$$3\theta = \frac{3\pi}{4} + 2k\pi \Rightarrow \theta_1 = \frac{\pi}{4}$$

$$\theta = \frac{\pi}{4} + n \cdot \frac{2\pi}{3}$$

$$\theta_2 = \frac{11\pi}{12}$$

$$\theta_3 = \frac{19\pi}{12}$$

Les solutions sont:

$$z_1 = \left[\frac{\sqrt{2}}{2}; \frac{\pi}{4} \right] = \frac{\sqrt{2}}{2} \left(\underbrace{\cos\left(\frac{\pi}{4}\right)}_{\frac{\sqrt{2}}{2}} + \underbrace{\sin\left(\frac{\pi}{4}\right)i}_{\frac{\sqrt{2}}{2}} \right) = \frac{1}{2} + \frac{1}{2}i$$

$$z_2 = \left[\frac{\sqrt{2}}{2}; \frac{11\pi}{12} \right] = \frac{\sqrt{2}}{2} \left(\cos\left(\frac{11\pi}{12}\right) + \sin\left(\frac{11\pi}{12}\right)i \right)$$

$$z_3 = \left[\frac{\sqrt{2}}{2}; \frac{19\pi}{12} \right] = \frac{\sqrt{2}}{2} \left(\cos\left(\frac{19\pi}{12}\right) + \sin\left(\frac{19\pi}{12}\right)i \right)$$

Remarquons que $z_1^4 = \left[\frac{4}{16}; \pi \right] = \left[\frac{1}{4}; \pi \right] = -\frac{1}{4}$

Exercise 5

$$z^3 = 2 - 2i = 2(1 - i) = 2 \left[\sqrt{2}; \frac{7\pi}{4} \right]$$

$$\text{Si } z = [r, \theta] \text{ alors } z^3 = [r^3, 3\theta]$$

$$r^3 = 2\sqrt{2} \Rightarrow r = \sqrt[3]{2\sqrt{2}} = \sqrt[3]{12}$$

$$3\theta = \frac{7\pi}{4} + 2k\pi$$

$$\theta = \frac{7\pi}{12} + k\frac{2\pi}{3}$$

$$z_1 = \left[\sqrt[3]{12}, \frac{7\pi}{12} \right], \quad z_2 = \left[\sqrt[3]{12}, \frac{5\pi}{4} \right], \quad z_3 = \left[\sqrt[3]{12}, \frac{23\pi}{12} \right]$$

Exercice 6

Formule de Moivre: $(\cos(\theta) + \sin(\theta)i)^5 = \cos(5\theta) + \sin(5\theta)i$

Après développement :

$$\cos(5\theta) = \cos^5(\theta) - 10\sin^2(\theta)\cos^3(\theta) + 5\sin^4(\theta)\cos(\theta)$$

$$\sin(5\theta) = \sin^5(\theta) + 5\sin(\theta)\cos^4(\theta) - 10\sin^3(\theta)\cos^2(\theta)$$

Exercice 7

$$z = 2 - i \quad \text{et} \quad w = 3 + 4i$$

Exercice 8

a) $z_1 = 2$, $z_2 = 12 + 5i$

b) $x_1 = 4 - i$, $x_2 = -4 + i$, $x_3 = 4 + i$, $x_4 = -4 - i$

c) $z_1 = i$ (double) , $z_2 = -2i$

d) $z_1 = -1 + i$, $z_2 = 3 - 2i$