Nombres complexes - série 3

Exercice 4

① Par Horner :

	i	$10+3i$	$25-8i$	$-14+7i$	$-99+207i$
$9i$		-9	$-27+9i$	$-9-18i$	$99-207i$
	i	$1+3i$	$-2+i$	$-23-11i$	0

② Par Horner :

	i	$1+3i$	$-2+i$	$-23-11i$
$2-i$		$1+2i$	$9+8i$	$23+11i$
	i	$2+5i$	$7+9i$	0

$(2+5i)(2-i) = 9+8i$

$(7+9i)(2-i) = 14+9+18i-7i = 23+11i$

③ L'équation devient :

$$(z-9i)(z-(2-i))(iz^2+(2+5i)z+7+9i)=0$$

On résout l'équation du 2ème degré :

$$iz^2+(2+5i)z+7+9i=0$$

$$\Delta = (2+5i)^2 - 4i(7+9i) = 4+25i^2+20i-28i-36i^2$$

$$= 15-8i = (a+bi)^2$$

$$\begin{cases} a^2-b^2=15 \\ a^2+b^2=17 \\ 2ab=-8 \end{cases} \quad \begin{array}{c|c} b & a \\ \hline \cdot 1 & \cdot(-1) \\ \cdot 1 & \cdot 1 \end{array} \quad \Leftrightarrow \begin{cases} 2a^2=32 \\ 2b^2=2 \\ 2b=-4 \end{cases} \Leftrightarrow \begin{array}{l} a=\pm 4 \\ b=\pm 1 \\ 2b=-4 \end{array}$$

On a : $4-i$ et $-4+i$.

Les deux solutions de l'équation de degré 2 :

$$z_1 = \frac{-(2+5i)+4-i}{2i} = \frac{2-6i}{2i} = \frac{1-3i}{i}\,\frac{-i}{-i} = -3-i$$

$$z_2 = \frac{-(2+5i)-4+i}{2i} = \frac{-6-4i}{2i} = \frac{-3-2i}{i}\,\frac{-i}{-i} = -2+3i$$

$$S = \{\, 9i \;;\; 2-i \;;\; -3-i \;;\; -2+3i \,\}$$

Exercice 3

$$z^3 + (1-3i)z^2 - (6-i)z + 10i = 0$$

① Soit $a \in \mathbb{R}$ la solution réelle. Alors

$$a^3 + (1-3i)a^2 - (6-i)a + 10i = 0$$
$$a^3 + a^2 - 3a^2 i - 6a + ai + 10i = 0$$
$$\Rightarrow (a^3 + a^2 - 6a) + (-3a^2 + a + 10)i = 0$$

$$\begin{cases} a(a^2 + a - 6) = 0 \\ 3a^2 - a - 10 = 0 \end{cases} \Leftrightarrow \begin{cases} a(a+3)(a-2) = 0 \\ (3a+5)(a-2) = 0 \end{cases}$$

$\Rightarrow a = 2$ est la solution réelle.

② Par Horner :

	1	$1-3i$	$-6+i$	$10i$
②		2	$6-6i$	$-10i$
	1	$3-3i$	$-5i$	0

On résout l'équation du 2ème degré :

$$z^2 + (3-3i)z - 5i = 0$$

$$\Delta = (3-3i)^2 - 4 \cdot (-5i) = 9 + 9i^2 - 18i + 20i = 2i$$

$$\begin{cases} a^2 - b^2 = 0 \\ a^2 + b^2 = 2 \\ 2ab = 2 \end{cases} \quad \begin{matrix} \cdot 1 \\ \cdot 1 \\ 1 \end{matrix} \quad \begin{matrix} \cdot(-1) \\ 1 \end{matrix} \quad \begin{cases} 2a^2 = 2 \\ 2b^2 = 2 \\ ab = 1 \end{cases} \Rightarrow \begin{cases} a = \pm 1 \\ b = \pm 1 \\ ab = 1 \end{cases}$$

On a $1+i$ et $-1-i$

Les solutions sont :

$$z_1 = \frac{-3+3i+1+i}{2} = \frac{-2+4i}{2} = -1+2i$$

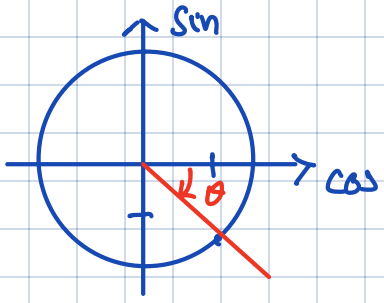
$$z_2 = \frac{-3+3i-1-i}{2} = \frac{-4+2i}{2} = -2+i$$

$$S = \{ 2 ; -1+2i ; -2+i \}$$

Problème 1

- $z_1 = \frac{1}{3} - \frac{1}{3}i$ $\quad |z_1| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \frac{1}{3}\sqrt{2} = \frac{\sqrt{2}}{3}$

argument : $\begin{cases} \cos(\theta) = \dfrac{1/3}{\sqrt{2}/3} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \\ \sin(\theta) = \dfrac{-1/3}{\sqrt{2}/3} = -\dfrac{\sqrt{2}}{2} \end{cases}$



$\theta = -\frac{\pi}{4}$

$$\boxed{z_1 = \frac{\sqrt{2}}{3} e^{i\frac{-\pi}{4}}}$$

- $z_2 = \underbrace{(1+i)}_{w}{}^6 = (\sqrt{2}e^{i\frac{\pi}{4}})^6 = \boxed{8 e^{i\frac{3\pi}{2}}}$

$|w| = \sqrt{2}$, $\theta = \frac{\pi}{4}$

- $z_3 = \frac{w_1}{w_2} = \frac{2+2i}{-\sqrt{3}+i} = \frac{\sqrt{8}\, e^{i\frac{\pi}{4}}}{2\, e^{-i\frac{\pi}{6}}} = \frac{\sqrt{8}}{2} e^{i\left(\frac{\pi}{4} - \left(-\frac{\pi}{6}\right)\right)}$

$= \frac{2\sqrt{2}}{2} e^{i\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)} = \boxed{\sqrt{2}\, e^{i\frac{5\pi}{12}}}$

Exercice 2

$w = 12 e^{-i\frac{\pi}{3}} = 12\left(\cos\left(-\frac{\pi}{3}\right) + \sin\left(-\frac{\pi}{3}\right)i\right)$

$= 12\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \underline{6 - 6\sqrt{3}i}$