15.09.26

**1.2.5** Calculer $z_1\, z_2$ et $z_1/z_2$ en utilisant la forme trigonométrique :

a) $z_1 = -1 + i, \quad z_2 = 1 + i$

b) $z_1 = -2 - 2\sqrt{3}\, i, \quad z_2 = 5\, i$

c) $z_1 = 2\, i, \quad z_2 = -3\, i$

d) $z_1 = -10, \quad z_2 = -4$

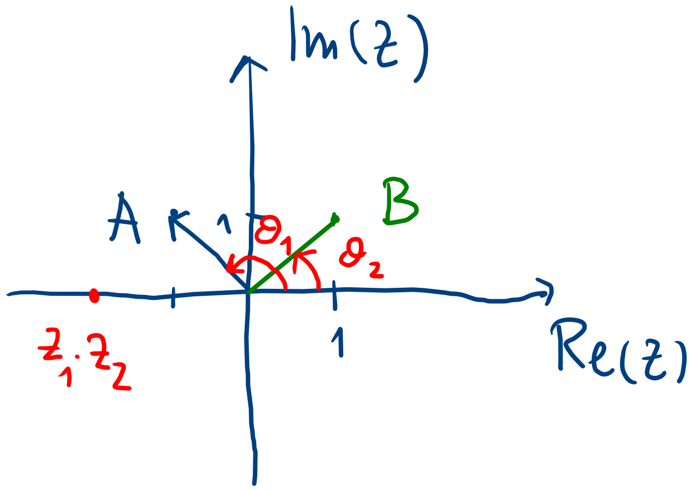
$\boxed{\pi \quad 180^\circ}$

$2\pi \quad 360^\circ$

$\frac{\pi}{2} \quad 90^\circ$

$\frac{\pi}{4} \quad 45^\circ$

a)




$z_1 = -1 + i \iff A(-1;\ 1)$

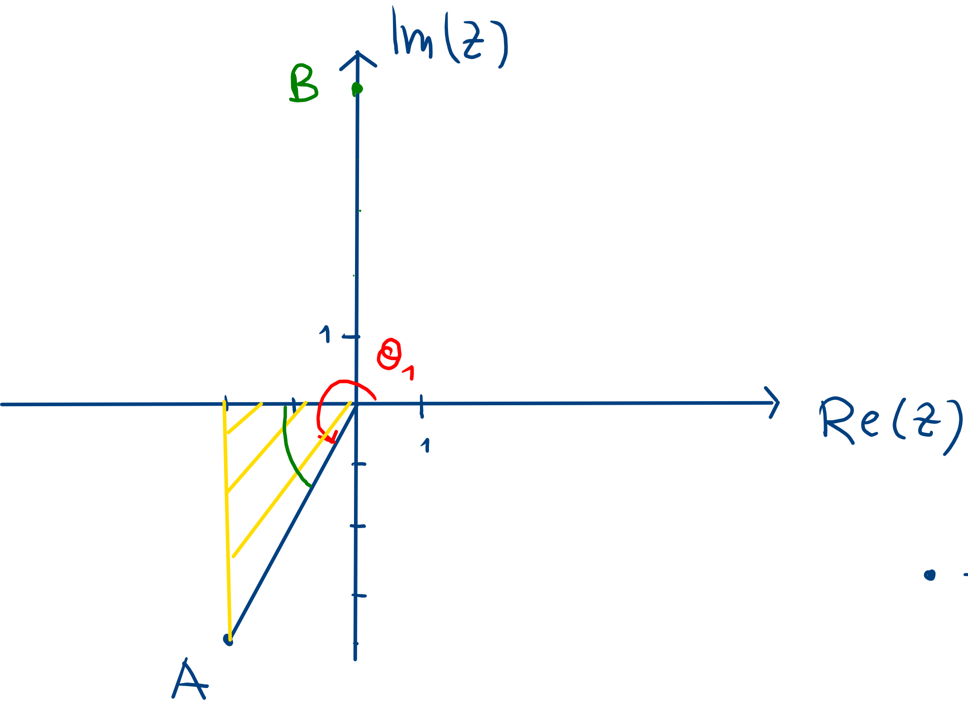
$z_2 = 1 + i \iff B(1;\ 1)$

$z_2 = \left[\sqrt{2}\ ;\ \frac{\pi}{4}\right]$ $\qquad z_1 = \left[\sqrt{2}\ ;\ \frac{3\pi}{4}\right]$

- $z_1 \cdot z_2 = \left[\sqrt{2}\cdot\sqrt{2}\ ;\ \frac{\pi}{4} + \frac{3\pi}{4}\right] = \left[2,\ \pi\right] = -2 = 2\left(\overbrace{\cos(\pi)}^{=-1} + \overbrace{\sin(\pi)}^{=0} i\right)$

- $\frac{z_1}{z_2} = \left[\frac{\sqrt{2}}{\sqrt{2}}\ ;\ \frac{3\pi}{4} - \frac{\pi}{4}\right] = \left[1;\ \frac{\pi}{2}\right] = 1\cdot\left(\overbrace{\cos\left(\frac{\pi}{2}\right)}^{=0} + \overbrace{\sin\left(\frac{\pi}{2}\right)}^{=1} i\right) = i$

b) $z_1 = -2 - 2\sqrt{3}\,i, \quad z_2 = 5\,i$




$z_1 = -2 - 2\sqrt{3}i \iff A(-2; -2\sqrt{3})$

$z_2 = 5i \iff B(0; 5)$

$240 \cdot \frac{\pi}{180} = \frac{\cancel{24}^{4}}{\cancel{18}_{3}}\pi$

$\bullet\ z_1 = [4; 240°] = \left[4; \frac{4\pi}{3}\right]$

$|z_1| = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = \sqrt{4+12} = 4$

$$\begin{cases} \cos(\theta_1) = \dfrac{Re(z_1)}{|z_1|} = \dfrac{-2}{4} = -\dfrac{1}{2} \\ \sin(\theta_1) = \dfrac{Im(z_1)}{|z_1|} = \dfrac{-2\sqrt{3}}{4} = -\dfrac{\sqrt{3}}{2} \end{cases}$$

TI
120°
-60°

$z_2 = \left[5; \frac{\pi}{2}\right]$

$z_1 \cdot z_2 = \left[4; \frac{4\pi}{3}\right] \cdot \left[5; \frac{\pi}{2}\right] = \left[20; \frac{11\pi}{6}\right] = 20\left(\cos\left(\frac{11\pi}{6}\right) + \sin\left(\frac{11\pi}{6}\right)i\right)$

$\frac{z_1}{z_2} = \left[\frac{4}{5}; \frac{4\pi}{3} - \frac{\pi}{2}\right] = \left[\frac{4}{5}; \frac{5\pi}{6}\right]$

# Application de la formule de Moivre

**1.2.7** Déterminer les formules de $\sin(4\theta)$ et $\cos(4\theta)$.

But : calculer $\sin(4\theta)$ en fonction de $\sin(\theta)$ et $\cos(\theta)$

CRM

$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) = 1 - 2\sin^2(\alpha) = 2\cos^2(\alpha) - 1$

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$

$\sin(4\theta) = ?$ et $\cos(4\theta) =$

$$(\cos(\theta) + \sin(\theta)\, i)^4 \overset{\text{Moivre}}{=} \cos(4\theta) + \sin(4\theta)\, i$$

$$= [1\,;\theta]^4 = [1^4\,; 4\theta] = [1, 4\theta]$$

Développons $(\cos(\theta) + \sin(\theta)\, i)^4$ avec le triangle de Pascal

$$(\cos(\theta) + \sin(\theta)\,i)^4 = \cos^4(\theta) + 4\cos^3(\theta)\sin(\theta)\, i + 6\cos^2(\theta)\sin^2(\theta)\, \underbrace{i^2}_{-1} +$$

$$+ 4\cos(\theta)\sin^3(\theta)\, \underbrace{i^3}_{-i} + \sin^4(\theta)\, \underbrace{i^4}_{1}$$

$$= \underbrace{\left(\cos^4(\theta) - 6\cos^2(\theta)\sin^2(\theta) + \sin^4(\theta)\right)}_{\cos(4\theta)} + \underbrace{\left(4\cos^3(\theta)\sin(\theta) - 4\cos(\theta)\sin^3(\theta)\right)}_{\sin(4\theta)} i$$

$(a+b)^n$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$(a+b)^0$

$(a+b)^1$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

**1.2.9** Déterminer sous forme trigonométrique et représenter dans le plan complexe :

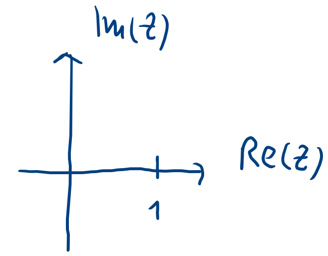
a) les racines septièmes de l'unité,

Déterminons $z \in \mathbb{C}$ tel que $z^7 = 1$.

Nous avons $z_1 = 1$. Déterminons $z_2, \dots, z_7 \in \mathbb{C}$ tels que $z_k^7 = 1$, $2 \leq k \leq 7$.

On pose $z = a + bi = [r, \theta]$

$$z^7 = [r^7 ; 7\theta] = [1 ; 0]$$




Donc $\begin{cases} r^7 = 1 \\ 7\theta = 0 + 2k\pi \end{cases}, k \in \mathbb{Z} \iff \begin{cases} r = 1 \\ \theta = 2k \cdot \frac{\pi}{7} \end{cases}, \quad k \in \mathbb{Z}$

$\underline{k=0}$ : $z_1 = [1 ; 0] = 1$

$\underline{k=1}$ $z_2 = [1 ; \frac{2\pi}{7}] = \cos\left(\frac{2\pi}{7}\right) + \sin\left(\frac{2\pi}{7}\right) i \cong 0{,}62349 + 0{,}78183\, i$

$\underline{k=2}$ $z_3 = [1 ; \frac{4\pi}{7}] =$ $z_C = -0.22252 + 0.97493i$

$\underline{k=3}$ $z_4 = [1 , \frac{6\pi}{7}] =$

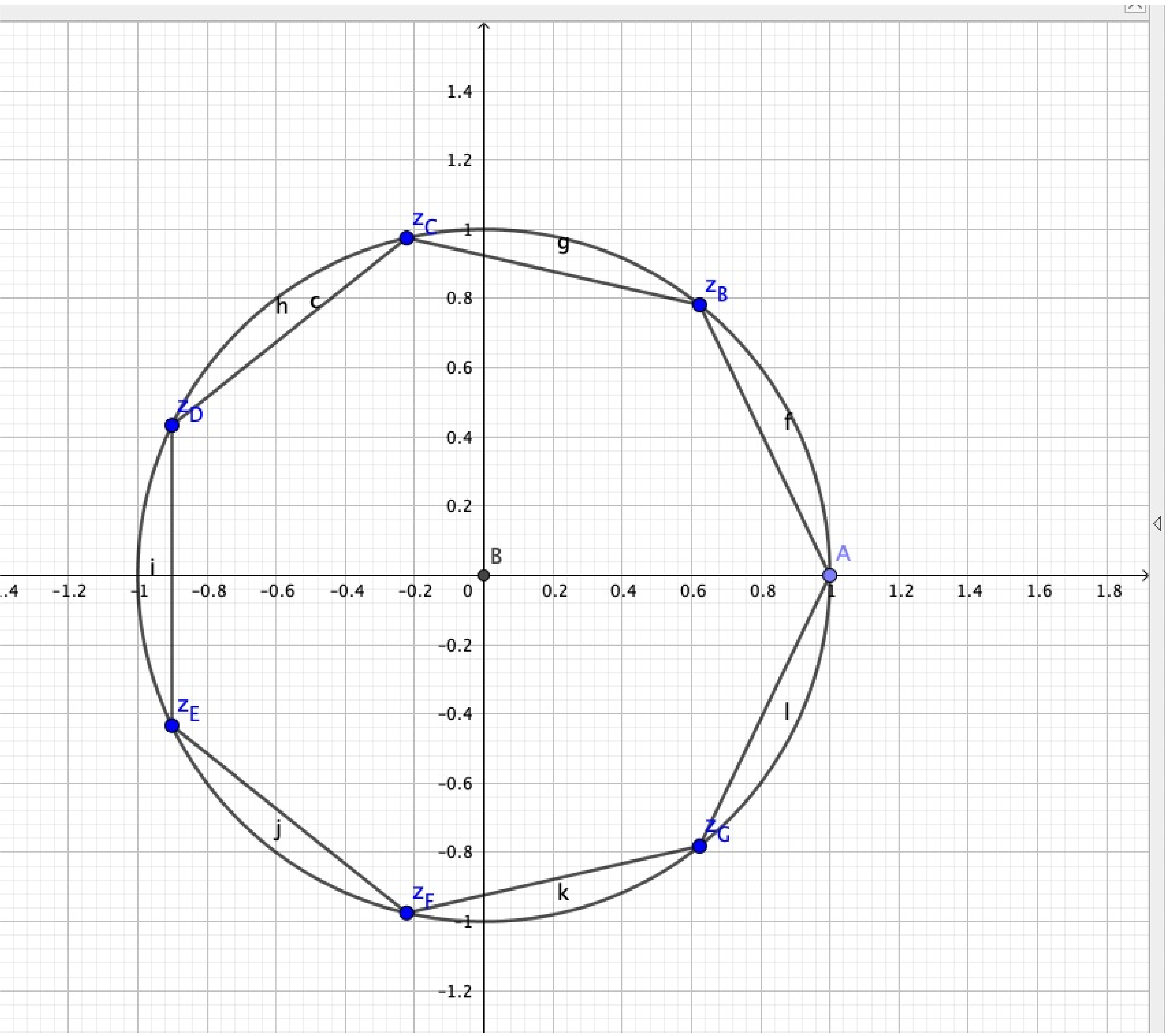
$\underline{k=4}$ $z_5 = [1 ; \frac{8\pi}{7}] =$

$\underline{k=5}$ $z_6 = [1 ; \frac{10\pi}{7}] =$

$\underline{k=6}$ $z_7 = [1 ; \frac{12\pi}{7}] =$

- $z_A = 1 + 0i$
- $A = (1, 0)$
- $z_B = 0.62349 + 0.78183i$
- $z_C = -0.22252 + 0.97493i$
- $z_D = -0.90097 + 0.43388i$
- $z_E = -0.90097 - 0.43388i$
- $z_F = -0.22252 - 0.97493i$
- $z_G = 0.62349 - 0.78183i$

En plançant les 7 points dans le plan, d'affixe $z_1$ à $z_7$,




On obtient un heptagone régulier inscrit dans un cercle de rayon 1

b) les solutions de l'équation $z^5 = -32$.