

1.3.2 Décomposer dans $\mathbb{R}[z]$ et $\mathbb{C}[z]$ les polynômes ci-dessous :

e) $z^3 - 6z^2 + 13z - 10$

$$p = z^3 - 6z^2 + 13z - 10$$

Si $w \in \mathbb{C}$ annule p , $\bar{w}$ annule aussi p

$$p(1) \neq 0, \quad p(-1) \neq 0, \quad p(2) = 0, \quad p(-2) =$$

$$z - 2 \mid p :$$

$$p = (z - 2) \underbrace{(z^2 - 4z + 5)}_{p_1} \in \mathbb{R}[z]$$

Factorisons p_1 : $z^2 - 4z + 5 = 0$

$$D = (-4)^2 - 4 \cdot 1 \cdot 5 = 16 - 20 = -4 = (2i)^2$$

$$z_1 = \frac{4 + 2i}{2} = 2 + i, \quad z_2 = \bar{z}_1 = 2 - i$$

$$p = (z - 2)(z - (2 + i))(z - (2 - i)) \in \mathbb{C}[z]$$

$$\text{f)} \quad z^6 + 9z^4 + 27z^2 + 27 = p$$

Posons $z^2 = w$

$$p_1 = w^3 + 9w^2 + 27w + 27 = (w+3)^3$$

$$p_1(1) \neq 0$$

$$p_1(-1) \neq 0$$

$$p_1(-3) = -27 + 81 - 81 + 27 = 0$$

Divisons p_1 par $w+3$:

1	9	27	27
-3	-3	-18	-27
1	6	9	0

$$p_1 = (w+3) \underbrace{(w^2 + 6w + 9)}_{(w+3)^2} = (w+3)^3$$

Donc

$$p = \underbrace{(z^2 + 3)^3}_{\in \mathbb{R}[z]} = \left[\underbrace{(z + \sqrt{3}i)(z - \sqrt{3}i)}_{\in \mathbb{C}[z]} \right]^3 = (z + \sqrt{3}i)^3 (z - \sqrt{3}i)^3$$

1.3.3 Résoudre dans $\mathbb{C}$ les équations ci-dessous :

a) $z^4 - (6 + 3i)z^3 + (8 + 12i)z^2 = 0$

b) $z^3 + 2z^2 + (-4 + 4i)z + 16 + 16i = 0$, sachant que -4 est un zéro

c) $z^3 - 4z^2 + (8 + i)z - 7 + i = 0$, sachant que $1 - i$ est un zéro

a) $z^2 \left(\underbrace{z^2 - (6 + 3i)z + (8 + 12i)} \right) = 0$

$$D = (6 + 3i)^2 - 4 \cdot (8 + 12i)$$

$$= 36 - 9 + 36i - 32 - 48i = -5 - 12i$$

Il faut trouver $d = a + bi$ tel que $d^2 = D$.

$$\begin{array}{l} \text{Re} \\ \text{module} \\ \text{Im} \end{array} \quad \begin{cases} a^2 - b^2 = -5 \\ a^2 + b^2 = 13 \\ 2ab = -12 \end{cases} \Leftrightarrow \begin{cases} a^2 = 4 \\ b^2 = 9 \\ 2ab = -6 \end{cases} \Leftrightarrow \begin{cases} a = \pm 2 \\ b = \pm 3 \\ ab = -6 \end{cases}$$

$$d_1 = 2 - 3i ; d_2 = -2 + 3i$$

$$z_1 = \frac{6 + 3i + 2 - 3i}{2} = 4 ; z_2 = \frac{6 + 3i - 2 + 3i}{2} = \frac{4 + 6i}{2} = 2 + 3i$$

$$S = \{ 0 ; 4 ; 2 + 3i \}$$

b) $z^3 + 2z^2 + (-4 + 4i)z + 16 + 16i = 0$, sachant que -4 est un zéro

Posons $p = z^3 + 2z^2 + (-4 + 4i)z + 16 + 16i$:

① Divisons p par $z + 4$ par Horner:

1	2	$-4 + 4i$	$16 + 16i$
-4	-4	8	$-16 - 16i$
1	-2	$4 + 4i$	0



$$p = (z + 4)(z^2 - 2z + 4 + 4i)$$

② Factorisons $p_1 = z^2 - 2z + 4 + 4i$:

$$D = 4 - 4(4 + 4i) = 4 - 16 - 16i = -12 - 16i$$

$$D = (\underbrace{a+bi}_a)^2 = -12 - 16i$$

$$\begin{array}{l} \text{Re} \\ \text{Module} \\ \text{Im} \end{array} \begin{cases} a^2 - b^2 = -12 \\ a^2 + b^2 = 20 \\ 2ab = -16 \end{cases} \Leftrightarrow \begin{cases} 2a^2 = 8 \\ 2b^2 = 32 \\ 2ab = -16 \end{cases} \Leftrightarrow \begin{cases} a^2 = 4 \\ b^2 = 16 \\ ab = -8 \end{cases} \Leftrightarrow \begin{array}{l} a = \pm 2 \\ b = \pm 4 \\ ab = -8 \end{array}$$

Donc $d_1 = 2 - 4i$ et $d_2 = -2 + 4i$

Les solutions : $z_1 = \frac{2 + 2 - 4i}{2} = 2 - 2i$ et $z_2 = \frac{2 - 2 + 4i}{2} = 2i$

$$p = (z + 4)(z - (2 - 2i))(z - 2i)$$

③ $S = \{-4, 2 - 2i, 2i\}$

c) $\underbrace{z^3 - 4z^2 + (8+i)z - 7+i}_{P(z)} = 0$, sachant que $1-i$ est un zéro

$P(z)$

$$\bullet P(1-i) \mid P(z) \Rightarrow (z - (1-i)) \mid P(z)$$

Par Horner :

	1	-4	8+i	-7+i
1-i	↓			
		1-i	-4+2i	7-i
	1	-3-i	4+3i	0



$$(-3-i)(1-i) = -4+2i$$

$$(4+3i)(1-i) = 7-i$$

$$(z - (1-i)) \left(\underbrace{z^2 - (3+i)z + 4+3i}_{D} \right) = 0$$

$$D = (3+i)^2 - 4(4+3i)$$

$$= 8+6i-16-12i = -8-6i$$

On cherche $d = a+bi$ tel que $d^2 = D$

$$\begin{array}{l} \text{Re} \\ \text{module} \\ \text{Im} \end{array} \begin{cases} a^2 - b^2 = -8 \\ a^2 + b^2 = 10 \\ 2ab = -6 \end{cases} \Leftrightarrow \begin{cases} a^2 = 1 \\ b^2 = 9 \\ ab = -3 \end{cases} \Leftrightarrow \begin{cases} a = \pm 1 \\ b = \pm 3 \\ ab = -3 \end{cases}$$

$$d_1 = 1-3i, \quad d_2 = -1+3i$$

$$z_1 = 1-i; \quad z_2 = \frac{3+i + 1-3i}{2} = \frac{4-2i}{2} = 2-i$$

$$z_3 = \frac{3+i - 1+3i}{2} = \frac{2+4i}{2} = 1+2i$$

$$S = \{ 1-i; 2-i; 1+2i \}$$