25.08.26

4.2.1 Résoudre les équations ci-dessous :

k) $3^{4x+2} - 36 \cdot 3^{2x+1} = -243$

l) $5 \cdot 5^{4x-7} - 120 \cdot 5^{2x-3} = 625$

k) Suite de jeudi 20.08.26

Effectuons un changement de variable : $3^{2x+1} = y$, $y>0$

On résout : $y^2 - 36y + 243 = 0$

Mardi : avec Δ...

On obtient $y = 27$ et $y = 9$

Revenons au changement de variables :

① $y = 9 \iff 3^{2x+1} = \underbrace{3^2}_{9} \iff 2x+1 = 2 \iff \boxed{x = \frac{1}{2}}$

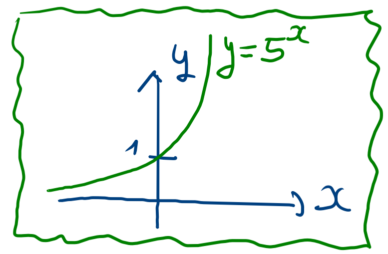
② $y = 27 \iff 3^{2x+1} = \underbrace{3^3}_{27} \iff 2x+1 = 3 \iff \boxed{x = 1}$

$$S = \left\{ 1 ; \frac{1}{2} \right\}$$

1) $5 \cdot 5^{4x-7} - 120 \cdot 5^{2x-3} = 625$

$$5 \cdot \underbrace{\left(5^{4x-6} \cdot 5^{-1}\right)}_{5^{4x-7}} - 120 \cdot 5^{2x-3} - 625 = 0$$

$$\underbrace{5^{2(2x-3)}}_{\left(5^{2x-3}\right)^2} - 120 \cdot \underbrace{\left(5^{2x-3}\right)}_{y>0} - 625 = 0$$



On résout $y^2 - 120y - 625 = 0$

$$(y-125)(y+5) = 0$$

$$y = 125 \quad \text{ou} \quad y = -5$$

① $y = -5$, pas de solution $\left[\,5^{2x-3} = -5\,\right]$

② $y = 125$, $5^{2x-3} = 5^3 \iff 2x-3 = 3$

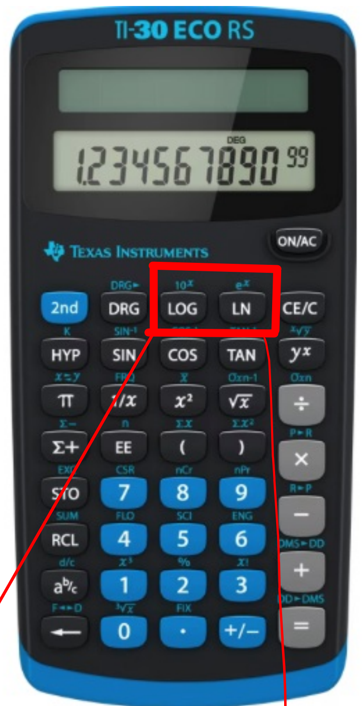
$$\iff \boxed{x = 3}$$

$$S = \{3\}$$

Les log

Résoudre : (1) $2^{x} = 8 \iff x = 3$

(2) $2^{x} = 10 \iff 3{,}2 < x < 3{,}4$ avec la $\boxed{TI}$



$\underbrace{LOG = \log}_{\log_{10}}$; $\underbrace{LN = \ln}_{\log_e}$

Soit $a > 0$, $a \neq 1$.

$$\boxed{a^{y} = x \iff y = \log_a(x)}$$

Exemples

- $\log_2(2) = 1 \iff 2^{1} = 2$
- $\log_2(8) = 3 \iff 2^{3} = 8$
- $\log_2(1024) = 10 \iff 2^{10} = 1024$
- $\log_2(0{,}125) = -3 \iff 2^{-3} = 0{,}125$
- $\log_{10}(2026) = \log(2026) = 3.3066394410243$

$\iff 10^{3.3066394410243} \simeq 2026$

4.2.2 Calculer à la main :

a) $\log_3(1) = 0$ b) $\log_2(8) = 3$ c) $\log_2(64) = 6$ d) $\log_2(1'024) = 10$

e) $\log_5(5) = 1$ f) $\log_3(\sqrt{3}) = \frac{1}{2}$ g) $\log_{243}(1/243) = -1$ h) $\log_3(27) = 3$

i) $\log(1'000) = 3$ j) $\log_4(\sqrt{2}) = \frac{1}{4}$ k) $\log_{1/8}(64) = -2$ l) $\log_5(0,04)$

m) $\log_3(\sqrt[4]{27}) = 3^{3/4}$ n) $\ln(e^2) = 2$ o) $\log_a(a) = 1$ p) $\log_a(a^3) = 3$

q) $\log(10000) = 4$ r) $\ln(e) = 1$ s) $\log_2(1/8) = -3$ t) $\log_3(\sqrt[4]{3}) = \frac{1}{4}$

u) $\log(200) - \log(2)$ v) $\log_6(4) + \log_6(9)$ w) $\log_5(1) = 0$ x) $\cancel{\log(-1)}$

y) $\log(0.0001) = -4$ z) $\cancel{\ln(0)}$

$$\boxed{\log_a(x) = y \iff a^y = x} \qquad a > 0, \quad a \neq 1$$

$$j) \quad 4^x = \sqrt{2} \iff 4^x = \underbrace{4^{\frac{1}{4}}}$$

$$\sqrt[4]{4} = \sqrt{\sqrt{4}} = \sqrt{2}$$

$$k) \quad \left(\frac{1}{8}\right)^x = 64 \iff \left(\frac{1}{8}\right)^x = 8^2$$

$$\left(\left(\frac{1}{8}\right)^{-1}\right)^2 = \left(\frac{1}{8}\right)^{-2}$$

$$\log_3(\sqrt[4]{27}) = \log_3(3^{3/4}) = \frac{3}{4}$$

Propriétés

Soit $a > 0$, $a \neq 1$. $\boxed{\log_a(y) = x \Leftrightarrow a^x = y}$

1) $a^{\log_a(y)} = y$, $y > 0$

2) $\log_a(a^x) = x$

3) $\log_a(1) = 0$

4) $\log_a(a) = 1$

Soit u, v tels que $\log_a(x) = u$ et $\log_a(y) = v$

On a $\log_a(x) = u \Leftrightarrow a^u = x$

et $\log_a(y) = v \Leftrightarrow a^v = y$

5) $\log_a(xy) = \log_a(a^u \cdot a^v) = \log_a(a^{u+v}) \overset{2)}{=} u + v$

$= \log_a(x) + \log_a(y)$

$5 = \log(100'000) = \log(100 \cdot 1000) = \log(100) + \log(1000)$

$= 2 + 3$

$$\boxed{\log_a(xy) = \log_a(x) + \log_a(y)}$$

6) $0 = \log_a(1) = \log_a\left(x \cdot \frac{1}{x}\right) \overset{5)}{=} \log_a(x) + \underbrace{\log_a\left(\frac{1}{x}\right)}_{-\log_a(x)}$

$$\boxed{\log_a\left(\frac{1}{x}\right) = -\log_a(x)}$$

7) $\log_a(x^r) = \log_a((a^u)^r) = \log_a(a^{ur}) \overset{2)}{=} u \cdot r = r \cdot \log_a(x)$

$$\boxed{\log_a(x^r) = r \cdot \log_a(x)}$$

4.2.3 Sachant que $\log(2) = 0.3010$ et $\log(3) = 0.4771$, calculer sans la calculatrice :

a) $\log(6)$ b) $\log(16)$ c) $\log(\sqrt{2})$ d) $\log(0,5)$ e) $\log(36)$ f) $\log\left(\frac{8}{27}\right)$

a) $6 = 2 \cdot 3$; $\log(6) = \log(2 \cdot 3) = \log(2) + \log(3)$

$= 0,3010 + 0,4771 = 0,7781$

b) $\log(16) = \log(2^4) = 4 \cdot \log(2) = 4 \cdot 0,3010 = 1,2040$

c) $\log(\sqrt{2}) = \log(2^{\frac{1}{2}}) = \frac{1}{2} \log(2) = 0,5 \cdot 0,3010 = 0,1505$

f) $\log\left(\frac{8}{27}\right) = \log\left(\frac{2^3}{3^3}\right) = \log(2^3 \cdot 3^{-3}) = \log(2^3) + \log(3^{-3})$

$= 3 \log(2) - 3 \log(3)$

ou $\log\left(\left(\frac{2}{3}\right)^{-3}\right) = 3 \log\left(\frac{2}{3}\right) = 3 \left[\log(2) - \log(3)\right]$