06.11.26

2.2.9 a) et b)

2.2.8

Fonctions trigonométriques de la moitié d'un arc

$\cos^2\left(\frac{\alpha}{2}\right) = \frac{1+\cos(\alpha)}{2}$	$\sin^2\left(\frac{\alpha}{2}\right) = \frac{1-\cos(\alpha)}{2}$
$\tan^2\left(\frac{\alpha}{2}\right) = \frac{1-\cos(\alpha)}{1+\cos(\alpha)}$	$\tan\left(\frac{\alpha}{2}\right) = \frac{1-\cos(\alpha)}{\sin(\alpha)} = \frac{\sin(\alpha)}{1+\cos(\alpha)}$

2.2.6 Calculer :

a) $\int \cos^2(x)\,dx = \int \frac{1+\cos(2x)}{2}\,dx = \frac{1}{2}\int (1+\cos(2x))\,dx$

$= \frac{1}{2}\left(\int 1\,dx + \int \cos(2x)\,dx\right) = \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + c$

$= \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$

c)

c) $\int \cos^3(x)\,dx = \int \cos^2(x)\cdot\cos(x)\,dx$

$= \int (1-\sin^2(x))\cos(x)\,dx = \int \cos(x)\,dx - \int \sin^2(x)\cos(x)\,dx$

$= \sin(x) - \frac{1}{3}\sin^3(x) + C$

$= \sin(x) - \frac{1}{3}(\sin(x))^3 + C$

$F(x) = \sin(x) - \frac{1}{3}(\sin(x))^3 + C$

$F'(x) = f(x) = \cos(x) - \sin^2(x)\cdot\cos(x)$

d) $\int \sin^4(x)\,dx = \int (\sin^2(x))^2\,dx = \int \left(\frac{1-\cos(2x)}{2}\right)^2 dx$

$= \frac{1}{4}\int (1 - 2\cos(2x) + \cos^2(2x))\,dx$

$= \frac{1}{4}\int dx - \frac{1}{2}\int \cos(2x)\,dx + \frac{1}{4}\int \cos^2(2x)\,dx$

$= \frac{1}{4}x - \frac{1}{4}\sin(2x) + \frac{1}{4}\cdot\left[\frac{4x + \sin(4x)}{8}\right] + c$

$$\int \cos^2(x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + c = \frac{2x + \sin(2x)}{4} + c$$

$$\int \cos^2(2x)\,dx = \int \cos^2(t)\cdot\frac{1}{2}\,dt = \frac{1}{2}\int \cos^2(t)\,dt = *$$

$t = 2x$

$dt = 2\,dx \Leftrightarrow \frac{1}{2}dt = dx$

$* = \frac{1}{2}\,\frac{2t + \sin(2t)}{4} + c = \frac{1}{8}\left[2t + \sin(2t)\right] + c$

$= \frac{1}{8}\cdot 4x + \frac{1}{8}\sin(4x) + c = \frac{1}{2}x + \frac{1}{8}\sin(4x) + c$

$= \frac{4x + \sin(4x)}{8} + c$

$$\frac{1}{4}x - \frac{1}{4}\sin(2x) + \frac{1}{4}\cdot\left[\frac{4x + \sin(4x)}{8}\right] + C$$

$$= \frac{8x - 8\sin(2x) + 4x + \sin(4x)}{32} + C$$

$$= \frac{12x - 8\sin(2x) + \sin(4x)}{32} + C$$

2.2.9 Trouver l'expression mathématique de la fonction f, sachant que :

a) $f'(x) = 3x^2 - 4$, $f(5) = 54$;

$$f(x) = x^3 - 4x + C$$

$$f(5) = 125 - 20 + C = 54$$

$$C = -51$$

$$f(x) = x^3 - 4x - 51$$

b) $f''(x) = (x+1)(x-2)$, $f(1) = 8$, $f'(0) = 37/6$;

① ②

$$f''(x) = x^2 - x - 2$$

$$f'(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + C_1$$

$$f(x) = \frac{1}{12}x^4 - \frac{1}{6}x^3 - x^2 + C_1 x + C_2$$

② $f'(0) = C_1 = \frac{37}{6}$

① $f(1) = \frac{1}{12} - \frac{1}{6} - 1 + \frac{37}{6} + C_2$

$$= \frac{1 - 2 - 12 + 74}{12} + C_2 \quad \Rightarrow \quad \frac{61}{12} + C_2 = 8$$

$$\Rightarrow \quad C_2 = \frac{96 - 61}{12} = \frac{35}{12}$$

$$f(x) = \frac{1}{12}x^4 - \frac{1}{6}x^3 - x^2 + \frac{37}{6}x + \frac{35}{12}$$

2.2.9 c)

2.2.8