11.11.26

2.2.11 Déterminer la fonction f sachant qu'elle admet pour asymptote la droite

① $x - 2y + 8 = 0$

et que

② $f''(x) = -\frac{8}{x^3} = -8x^{-3}$

② $f'(x) = \int -8x^{-3}\,dx = -8\int x^{-3}\,dx = -8 \cdot \frac{1}{-2} x^{-2} + C_1$

$= 4x^{-2} + C_1$

$f(x) = \int 4x^{-2} + C_1\,dx = 4 \cdot \frac{1}{-1} x^{-1} + C_1 x + C_2$

$= -\frac{4}{x} + C_1 x + C_2$

$= \frac{C_1 x^2 + C_2 x - 4}{x}$

① $x - 2y + 8 = 0$ est une AO

$y = \frac{1}{2}x + 4$

$= C_1 x + C_2 - \frac{4}{x}$

$\uparrow$

$C_1 = \frac{1}{2} \qquad C_2 = 4$

$f(x) = \frac{\frac{1}{2}x^2 + 4x - 4}{x}$

2.2.12 Calculer :

a) $\int_1^4 (x^2 - 2x + 3)\, dx$

b) $\int_{-1}^1 (2x^3 + 3x^2 + 2x - 1)\, dx$

$$\int f(x)\,dx = F(x) + C$$

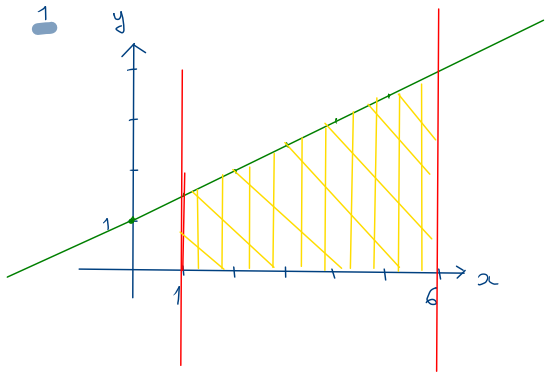
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

a) $\int_1^4 (x^2 - 2x + 3)\, dx = \frac{1}{3}x^3 - x^2 + 3x \Big|_1^4$

$= \left(\frac{64}{3} - 16 + 12\right) - \left(\frac{1}{3} - 1 + 3\right) = 21 - 4 - 2 = \underline{15}$

Intégrales définies : quelques propriétés

① $\int_1^6 \left(\frac{1}{2}x + 1\right) dx = \mathcal{A}$



$f(x) = \frac{1}{2}x + 1$

$\mathcal{A}$ est l'aire délimitée par

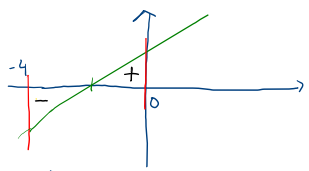
- La droite $x = 1$
- La droite $x = 6$
- La courbe $y = \frac{1}{2}x + 1$
- L'axe Ox

(i) $\mathcal{A} = \frac{f(1) + f(6)}{2}(6-1) = \frac{\frac{3}{2} + 4}{2} \cdot 5 = \frac{55}{4}$ (aire d'un trapèze)

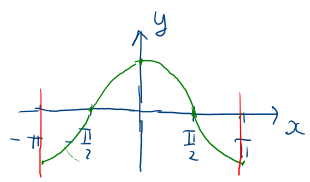
(ii) $\mathcal{A} = \int_1^6 \left(\frac{1}{2}x + 1\right) dx = \frac{1}{4}x^2 + x \Big|_1^6 = (9+6) - \left(\frac{1}{4} + 1\right)$

$= 15 - \frac{5}{4} = \frac{55}{4}$

② $\int_{-4}^{0} \left(\frac{1}{2}x + 1\right) dx = \frac{1}{4}x^2 + x \Big|_{-4}^{0} = 0 - 0 = 0$



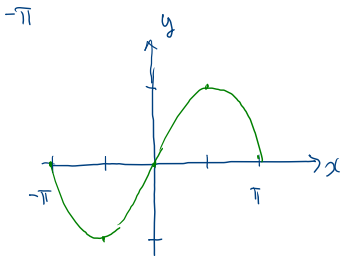
③ $\int_{-\pi}^{\pi} \cos(x)\, dx = \sin(x) \Big|_{-\pi}^{\pi} = \sin(\pi) - \sin(-\pi) = 0$



$\cos(-\alpha) = \cos(\alpha)$

④ $\int_{-\pi}^{\pi} \sin(x)\, dx = -\cos(x) \Big|_{-\pi}^{\pi} = -\left(\cos(\pi) - \cos(-\pi)\right) = -\left(1 - (-1)\right) = 0$

$= -\left(\cos(\pi) - \cos(\pi)\right) = 0$

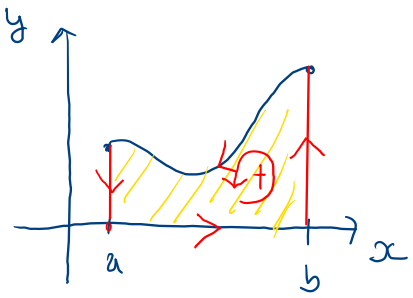


Propriétés de l'intégrale définie

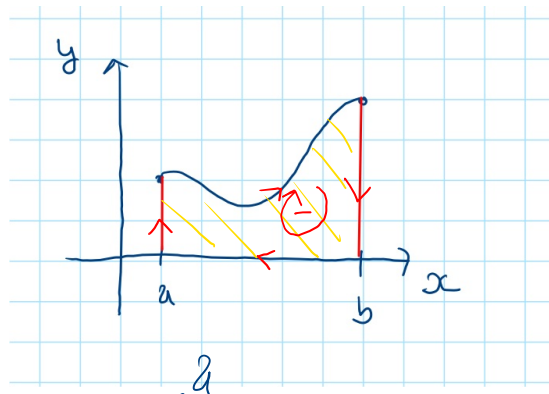
Soit f une fonction intégrable et continue sur un intervalle $[a,b]$. Soit F une primitive de f

① $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

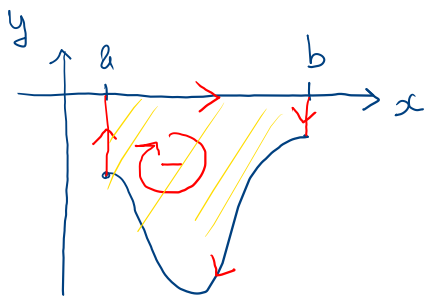
$$\int_a^b f(x)\,dx = F(b) - F(a) = -\big(F(a) - F(b)\big) = -\int_b^a f(x)\,dx$$



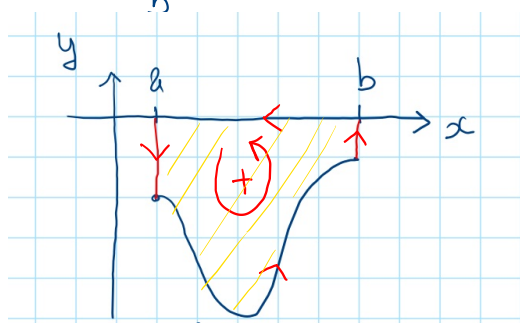
$$\int_a^b f(x)\,dx > 0$$



$$\int_b^a f(x)\,dx < 0$$



$$\int_a^b f(x)\,dx < 0$$



$$\int_b^a f(x)\,dx > 0$$