Vendredi 13.11.26

2.2.12 Calculer :

h) $\int_0^{\frac{\pi}{2}} \sin^2(x)\cos(x)\,dx$

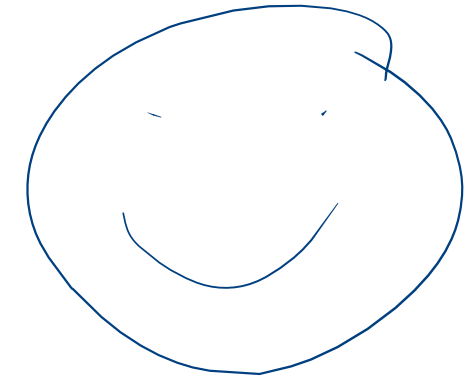
$= \int_0^1 t^2\,dt$

$t = \sin(x)$

$dt = \cos(x)\,dx$

x	t
$\frac{\pi}{2}$	1
0	0

$= \left[\frac{1}{3}t^3\right]_0^1 = \frac{1}{3}$.



g) $\int_1^4 \frac{dx}{\sqrt{x}}$

$$= \int_1^4 x^{-\frac{1}{2}}\,dx = 2\sqrt{x}\Big|_1^4$$

$$= 4 \cdot 2 - 2 \cdot 1$$

$$= 2$$

2.2.13 Sachant que $\int_0^1 f(x)\,dx = 3$, $\int_1^2 f(x)\,dx = 4$ et $\int_2^3 f(x)\,dx = -8$, calculer :

a) $\int_0^2 f(x)\,dx = 3 + 4 = 7$

b) $\int_0^1 3f(x)\,dx = 3 \cdot 3 = 9$

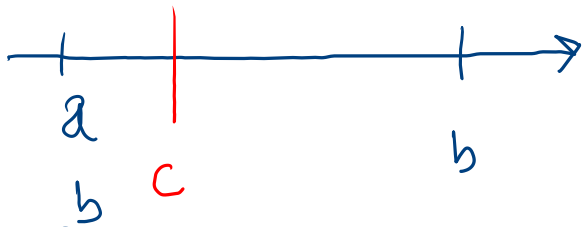
c) $\int_0^3 8f(x)\,dx = 8 \cdot [3 + 4 - 8] = -8$

d) $\int_3^1 2f(x)\,dx = 2\int_3^1 f(x)\,dx = -2\int_1^3 f(x)\,dx =$

$= -2[4 - 8] = 8$

① $a < c < b$:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



② $$\int_a^b K \cdot f(x)\,dx = K \cdot \int_a^b f(x)\,dx$$

③ $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

2.2.15 Déterminer les réels k pour lesquels on a :

a) $\displaystyle\int_{-1}^{2} kx^2\,dx = \frac{2}{3}$

c) $\displaystyle\int_{0}^{k/2} \cos(t)\,dt = \frac{1}{2}$

a) $f(x) = Kx^2$

$$F(x) = \int Kx^2\,dx = K\int x^2\,dx = K\cdot\frac{1}{3}x^3 + C$$

$$\int_{-1}^{2} Kx^2\,dx = K\left[\frac{8}{3} - \left(-\frac{1}{3}\right)\right] = 3K$$

D'où l'équation $3K = \frac{2}{3} \Leftrightarrow K = \frac{2}{9}$

b) $\int_4^k (x^2 - 3x + 7)\,dx = \frac{129}{2}$

$$\int (x^2 - 3x + 7)\,dx = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 7x + c$$

$$\int_4^k (x^2 - 3x + 7)\,dx = \left(\frac{1}{3}k^3 - \frac{3}{2}k^2 + 7k\right) - \left(\underbrace{\frac{64}{3} - 24 + 28}\right)$$

$$= \frac{2k^3 - 9k^2 + 42k}{6} - \frac{76}{3}$$

$$= \frac{2k^3 - 9k^2 + 42k - 152}{6}$$

D'où l'équation : $2k^3 - 9k^2 + 42k - 152 = 387$

$$\underbrace{2k^3 - 9k^2 + 42k - 539}_{p} = 0$$

$\frac{539}{2}$

$539 = 7 \cdot 77$ $\quad p(7) = 0$

Par Horner :

	2	−9	42	−539
×7		14	35	539
	2	5	77	0

L'équation : $(k-7)(\underbrace{2k^2 + 5k + 77}) = 0$

$\Delta < 0$

pas de solution

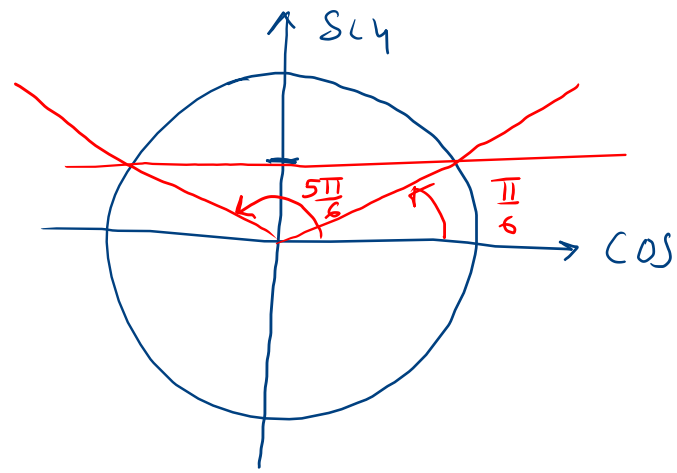
On obtient $k = 7$.

c) $\int_0^{k/2} \cos(t)\,dt = \frac{1}{2}$

$$\sin(t)\Big|_0^{k/2} = \frac{1}{2}$$

$$\sin\left(\frac{k}{2}\right) = \frac{1}{2}$$

c'est une équation trigonométrique



$\boxed{\text{TI}}$ $\frac{k}{2} = 30° = \frac{\pi}{6}$

- $\frac{k}{2} = \frac{\pi}{6} + 2k\pi$

$k = \frac{\pi}{3} + k \cdot 4\pi$

- $\frac{k}{2} = \frac{5\pi}{6} + k \cdot 2\pi$

$k = \frac{5\pi}{3} + k \cdot 4\pi$

2.2.17 Calculer les intégrales suivantes à l'aide du changement de variable indiqué :

a) $\int_{-2}^{0} x\sqrt{x+2}\,dx, \quad x = t^2 - 2$

$f(x) = x\sqrt{x+2}$, $ED(f) = [-2, +\infty[$

Changement de variables $x = t^2 - 2$

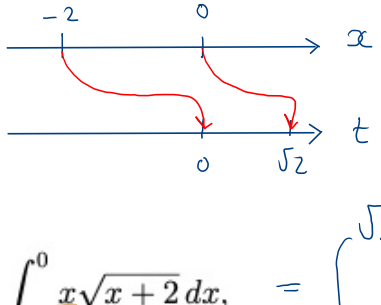
$dx = 2t\,dt$

x	t
-2	0
0	$\sqrt{2}$

Trouver les bornes : $x + 2 = t^2$

(i) $x = -2 \Rightarrow -2 + 2 = t^2 \Leftrightarrow t = 0$

(ii) $x = 0 \Rightarrow 2 = t^2 \Rightarrow t = \sqrt{2}$ ☺ ~~$-\sqrt{2}$~~



a) $\int_{-2}^{0} x\sqrt{x+2}\,dx, = \int_{0}^{\sqrt{2}} (t^2-2)\sqrt{t^2}\cdot 2t\,dt$

$= \int_{0}^{\sqrt{2}} (t^2-2)\cdot t\cdot 2t\,dt$

$= \int_{0}^{\sqrt{2}} (2t^4 - 4t^2)\,dt = \left[\frac{2}{5}t^5 - \frac{4}{3}t^3\right]_0^{\sqrt{2}}$

$= \left[\frac{6t^5 - 20t^3}{15}\right]_0^{\sqrt{2}} = \frac{1}{15}\left[6t^5 - 20t^3\right]_0^{\sqrt{2}}$

$= \frac{1}{15}\left[6\underbrace{(\sqrt{2})^5}_{4\sqrt{2}} - 20\underbrace{(\sqrt{2})^3}_{2\sqrt{2}}\right] = \frac{1}{15}\left[24\sqrt{2} - 40\sqrt{2}\right]$

$(\sqrt{2})^5 = \sqrt{2\cdot 2\cdot 2\cdot 2}\cdot\sqrt{2} = 4\sqrt{2}$ $\Big\}$ $= \frac{-16}{15}\sqrt{2}$